989 (Pages : 3) Name....................................

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4C

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2014

(CUCSS)

Mathematics

MT 2C 06—ALGEBRA—II

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions* (1 – 14)
Each question has weightage 1.

1. Verify whether {0, 3} is an ideal of Z_6.
2. Verify whether $\{(0,2n) : n \in z\}$ if a prime ideal of $Z \times Z$.
3. Verify whether the field Z_5 is an extension of the field Z_3.
4. Verify whether $Q(\sqrt{2})$ is an algebraic extension of Q.
5. Which of the following real numbers is constructible $\sqrt{2}, \sqrt[3]{2}, \sqrt[3]{5}, \sqrt[4]{3}$.
6. Verify whether $\varphi : Q(\sqrt{2}) \to Q\sqrt{3}$ defined by $a + b\sqrt{2} \mapsto a + b\sqrt{3}$ for $a, b \in Q$ is an isomorphism of fields.
7. Show that the field R of reals is not algebrically closed.
8. Let α be the real cube root of 2. Verify that $Q(\alpha)$ is not a splitting field.
9. Find the order of the group $G(Q(\alpha)/(Q)$ where α is the real cube root of 2.
10. Give an example of an infinite field of characteristic 2.
11. Describe the Galois group $G\left(Q(\sqrt{2})/Q\right)$.
12. Let K be a finite field of 8 elements and $F = Z_2$. Give the order of the Galois group G (K/F).
13. Define the n^{th} cyclotomic polynomial.
14. Give an example of a solvable group.

(14 × 1 = 14 wrightage)

Turn over

Part B

Answer any **seven** *questions from the following questions (15 – 24).*
Each question has weightage 2.

15. Find all prime ideals of the ring Z_8.

16. Let R be a ring with identity. Show that the map $\varphi : Z \to R$ defined by $\varphi(n) = n \cdot \perp$ homomorphism.

17. Let $p(x) = x^2 + 1 \in Q[x]$. Let $I = < p(x) >$ be the ideal generated by $p(x)$. Show that $x + I$ is a of $p(x)$ in $Q[x]/I$.

18. Let $f(x) = x^4 - \perp \in Q[x]$. Let $\alpha \notin Q$ be a zero of $f(x)$. Find the degree $[Q(\alpha) : Q]$.

19. Let α be a zero of $x^2 + 1 \in Z_3[x]$. Find the number of elements in $Z_3(\alpha)$.

20. Let σ be a automorphism of $Q(\sqrt{2}, \sqrt{3})$ with $\sigma(\sqrt{2}) = -\sqrt{2}$ and $\sigma(\sqrt{3}) = \sqrt{3}$. Find the f field of σ.

21. Let $f(x) \in Q[x]$ be irreducible and α, β be zeros of $f(x)$ in $\bar{Q}$. Let τ be an automorphism o Such that $\tau(\alpha) = \beta$. Let $\tau_x : \bar{Q}[x] \to \bar{Q}[x]$ be the natural isomorphism with $\tau_x(x) = x$. Show $\tau_x(f(x)) = f(x)$.

22. Let $F \leq E \leq K$ and K be a finite normal extension of a filed F. Show that K is a normal exten of E.

23. Let K be a field of a 9 elements and let $F = Z_3$. Show that $\sigma : K \to K$ defined by $\sigma(a) = a^3$ $a \in K$ is an automorphism of K leaving F fixed.

24. Let K be the splitting field of $x^4 + \perp$ over Q. Show that G (K/Q) is of order 4.

(7 × 2 = 14 weighta

Part C

Answer any **two** *questions from the following questions (25-28).*
Each question has weightage 4.

25. Define maximal ideal. Show that if R is a commutative ring with identity and if M is a maxi ideal of R then R/M is a field. Give an example of a commutative ring R with identity, a maxi ideal M of R and describe the field R/M.

6. Let E be an extension of a field F and let $\alpha \in E$. Prove that

(a) $\varphi_\alpha : F[x] \to E$ defined by $f(x) \mapsto f(\alpha)$ for $f(x) \in F[x]$, is a homomorphism.

(b) If α is algebraic over F then Ker $\varphi_\alpha \neq (0)$.

(c) If α is transcendental over F then φ_α is one-to-one.

7. Define separable extension. Show that every finite extension of a field of characteristics zero is a separable extension.

8. Define normal extension. Let be a field and $F \leq E \leq K \leq \bar{F}$. Show that if K is a normal extension of F then K is a normal extension of E.

Show that G(K/E) is a subgroup of G (K/F).

(2 × 4 = 8 weightage)