15U113

Name.....

Reg. No.....

FIRST SEMESTER DEGREE EXTERNAL EXAMINATION DECEMBER/ JANUARY 2015 – '16

(2015 Admission)

CC15UMAT1C01-MATHEMATICS (Complementary)

Time: 3 hrs **Max Mark: 80**

I Answer all questions **(12 x 1 = 12 *Marks*)**

1. Find $\lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2}$
2. Find a point of discontinuity of the function $f(x) = \frac{x+2}{\cos x}$
3. Find $\lim_{x \to \infty} \frac{\sin 2x}{x}$
4. If $\lim_{x \to 4} \frac{f(x)-5}{x-2} = 1$, find $\lim_{x \to 4} f(x)$
5. Find the point of inflexion of the curve $y = (x-2)^3 + 1$
6. Evaluate $\sum_{k=1}^{7}(-3k)$
7. Find the Norm of the partition { -2, -1.6, -0.5, 0, 0.8, 1} of the interval $[-2, 1]$
8. Find the vertical asymptote of the curve $y^2(2a - x) = x^3$
9. Find the slope the curve $f(x) = x^2 + 1$ *at* $(2, 5)$
10. If f is continuous and $\int_1^2 f(x)\,dx = -4$ and $\int_1^5 f(x)dx = 6$, then $\int_2^5 f(x)dx =$ .....
11. If f is integrable on $[a, b]$, the average value $AV(f)$ = ...
12. Express $\lim_{\|P\| \to 0} \sum_{K=1}^{n}(3c_k^2 - 2c_k + 5)\,\Delta x_k$ where P denotes a partition of the $[-1, 3]$, as an integral.

II Answer any nine questions **(9 x 2 = 18 *Marks*)**

13. Using $\varepsilon - \delta$ definition prove that $\lim_{x \to 3}(3x - 7) = 2$
14. Show that if $\lim_{x \to c}|f(x)| = 0$, then $\lim_{x \to c} f(x) = 0$
15. Evaluate $\lim_{x \to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$
16. Find the absolute extrema of $y = 4 - x^2$ on [-3, 1]
17. Find the linearization of $f(x) = \sqrt{x^2 + 9}$ at $x = -4$
18. For what value of a, the following function is continuous at $x = 3$,

$$f(x) = \begin{cases} x^2 - 2; & x < 3 \\ 2ax; & x \geq 3 \end{cases}$$

19. Use the inequality $cosx \geq 1 - \frac{x^2}{2}$, to find a lower bound for the value of $\int_0^1 cosx\, dx$

20. Find $\lim_{x \to 0} (1 + x)^{1/x}$

21. Find the area of the region between the curve $y = 3x^2$ and the x axis on the interval $[0, l$

22. Find the second derivative of the function $r = \frac{12}{\theta} - \frac{4}{\theta^2} + \frac{1}{\theta^4}$ w.r.t θ

23. Find $\int_{-4}^{4} |x|\, dx$

24. Suppose that $\int_0^1 f(x)dx = 3$, then if f is odd , find $\int_{-1}^{0} f(x)dx$

III *Answer any six questions* **(6 x 5 = 30 Mark**

25. Prove that the function $y = |x|$ is differentiable on (-∞, 0) and (0, ∞), but has no derivati at $x = 0$

26. In the alternate form of the Mean value theorem $f(a + h) = f(a) + hf^I(a + \theta h)$; $0 < \theta < 1$, If $f(x)$ is a quadratic expression, prove that $\theta = 1/2$

27. The curves $y = x^2 + ax + b$ and $y = cx - x^2$ have a common tangent line at the point (1, 0). Find a, b *and* c

28. Show that the function $f(x) = \begin{cases} 1 & \text{when } x \text{ is rational} \\ 0 & \text{when } x \text{ is irrational} \end{cases}$ is not Riemann integrable over [(

29. Find all the asymptotes of the curve $y = \frac{x^2 - 4}{x - 1}$

30. Verify Rolle's theorem for the function $f(x) = x^3 - 3x^2 + 2x$

31. Prove that the function $y = \tan\theta$ increases on every interval in its domain

32. Show that the value of $\int_0^1 \sqrt{1 + cosx}\, dx$ cannot possibly be 2

33. Estimate the sum of the square roots of the first n positive integers

IV Answer any two questions **(2 x 10 = 20 Mark**

34. Prove that the function $f(x) = \begin{cases} \frac{|x-a|}{x-a} & x \neq a \\ 1 & x = a \end{cases}$ *is* discontinuous *at* $x = a$. Is the discontinuity removable? If so what should $f(a)$ be? If not, why not?