SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2014

(U.G.—CCSS)

Complementary Course

MM 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

Unit I

Answer all **twelve** *questions.*
Each question carries ¼ weightage.

1. Tanh x is equal to :

   (a) $\dfrac{e^x - e^{-x}}{2}$.

   (b) $\dfrac{e^x + e^{-x}}{2}$.

   (c) $\dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

   (d) $\dfrac{e^x + e^{-x}}{e^x - e^{-x}}$.

2. Write $csch^{-1}x$ as a logarithmic function :

3. Find $\dfrac{d}{dx}\left(6\sinh\dfrac{x}{3}\right)$.

4. Find $\displaystyle\int_0^1 \dfrac{1}{\sqrt{x}}\,dx$.

5. The nth term of the sequence

   0, 3, 8, 15, 24 ......

6. Find $\displaystyle\lim_{n \to \infty} \sqrt[n]{3\,n}$.

7. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{3^{n-1}-1}{6^{n-1}}$.

8. Test the convergence of the series

   $1 + \dfrac{1}{3} + \dfrac{1}{7} + \dfrac{1}{15} + ...$

Turn over

9. Define the absolute convergence of a series Σa_n.

10. Find the Cartesian equation of the curve $r = 6\sin\theta$.

11. Write the polar equation of the hyperbola with $k = 2$ and $e = \frac{3}{2}$.

12. $f(x, y) = 100 - x^2 - y^2$. Find the level curve of $f(x, y) = 75$.

(12 × ¼ = 3 weighta

Unit II

*Answer any **nine** questions.*
Each question carries 1 weightage.

13. Differentiate $\tanh\sqrt{1 + t^2}$ with respect to t.

14. Find $\int \text{sech}^2\left(x - \frac{1}{2}\right)dx$.

15. Find $\int_0^2 \frac{dx}{1 - x^2} dx$.

16. Find the sum of the series $\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n)}$.

17. Does the series $1 + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + ...$ converge ?

18. Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ is absolutely convergent.

19. For what value of x does the power series $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{x^n}{n}$ converge ?

20. Find the Taylor series expansion of $f(x) = \ln(1 + x)$ at $x = 0$.

21. Find the polar equation of the elliptic $4x^2 + 9y^2 = 36$.

22. Write the centre and radius of the circle $r + \cos \theta = 0$.

23. Find $\displaystyle \lim_{p \to (0, -2, 0)} \left(\ln \sqrt{x^2 + y^2 + z^2} \right)$.

24. Find $\dfrac{\partial f}{\partial x}$ at $(4, -5)$ if $f(x, y) = x^2 + 3xy + y$.

(9 × 1 = 9 weightage)

Unit III

*Answer any **five** questions.*
Each question carries 2 weightage .

25. Show that $\cosh^{-1} x = \ln \left(x + \sqrt{x^2 - 1} \right)$, $x \geq 1$.

26. Find $\displaystyle \int_{-1}^{\infty} \dfrac{d\theta}{\theta^2 + 5\theta + 6}$.

27. Show that $\dfrac{1 + 2 \ln 2}{9} + \dfrac{1 + 3 \ln 3}{14} + \dfrac{1 + 4 \ln 4}{2} + \ldots\ldots$ diverges.

28. Find the points of intersection of the pair of curves $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$.

29. If $f(x, y) = x \cos y + y e^x$ find $\dfrac{\partial^2 f}{\partial x dy}$ and $\dfrac{\partial^2 f}{\partial y^2}$.

30. Express $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$ interms of r and s where $w = x^2 + y^2$, $x = r + s$, $y = r - s$.

31. Find the directional derivative of $f(x, y) = 3xy - y^2$ at $(5, 5)$ with directions of $\overline{A} = 4i + 3j$.

32. Find the area that lies inside the circle $r = 1$ and outside $r = 1 - \cos \theta$.

(5 × 2 = 10 weightage)

Turn over

Unit IV

*Answer any **two** questions.*
Each question carries 4 weightage.

33. (a) Evaluate $\int_{2}^{\infty} \dfrac{x+3}{(x-1)\left(x^2+1\right)}\, dx.$

(b) Investigate the convergence of $\displaystyle\sum_{n=1}^{\infty} \dfrac{4^n \left(n!\right)^2}{(2n)!}.$

34. Find the length of the cardioid $r = 1 - \cos\theta.$

35. Find the linearization of $f(x, y) = x^2 - xy + \dfrac{y^2}{2} + 3$ at the point $(3, 2).$

(2 × 4 = 8 weightage)