16U112

FIRST SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2016

(Regular/ Supplementary/ Improvement)

(CUCBCSS - UG)

CC15UMAT1B01– FOUNDATIONS OF MATHEMATICS

(Mathematics Core course)

(2015 Admission onwards)

Time: Three Hours   Maximum: 80 Marks

I.   **Answer *all* questions:**

1. Find the domain of $f(t) = \dfrac{1}{\sqrt{t}}$.

2. The range of the function $y = x^2$ is.............

3. Let $\lfloor x \rfloor$ denote the integer floor function at x then $\lfloor -3.2 \rfloor$ =......

4. $\varphi \times A$ = ...

5. The graph of $y = (x-3)^3 - 2$ is the graph of $y = x^3$ shifted ....

6. What is meant by 'the power of continuum'.

7. Is $1+ 3 = 6$ a proposition?

8. State true or false ." If a relation R is symmetric, then R^C is symmetric ."

9. Let $S = \{1,2,3,4,5,6,7,8\}$. Write down a partition of S .

10. Let A= $\{1,2,3,4\}$ and R = $\{(1,2),(2,3),(3,1)\}$. Then R^2 = ...

11. Give an example of a poset.

12. The points of discontinuity of the function $f(x) = \dfrac{\cos x}{x+2}$ is ....

$(12 \times 1 = 12 \text{ Marks})$

II.   **Answer any nine questions.**

13. Define characteristic function of a set A.

14. Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not equivalent.

15. Find $\lim\limits_{x \to -2} \dfrac{-2x-4}{x^3 + 2x^2}$.

16. Graph the circle $x^2 + y^2 - 4x - \dfrac{9}{4} = 0$

17. Prove that the function $f(x) = \sqrt{4 - x^2}$ is continuous on [-2, 2].

18. Show that $\lim\limits_{x \to c} | f(x)| = 0$ then $\lim\limits_{x \to c} f(x) = 0$

19. Establish Intermediate Value Theorem.

20. Prove that the relation $x \equiv y \pmod 5$ on the set of integers is an equivalence relation.

21. Let A= $\{1,2,3,4\}$ Draw the directed graph of the relation
    R= $\{(1,1),(2,2),(2,3),(3,2),(4,2),(4,4)\}$.

22. Let X=$\{1,2,3….9\}$ Find the cross partition of the following partitions of X
    $P_1 = [\{1,3,5,7,9\},\{2,4,6,8\}]$

    $P_2 = [\{1,2,3,4\},\{5,7\},\{6,8,9\}]$

23. Verify that $p \vee p \equiv p$ and $p \wedge p \equiv p$.

24. If R is a partial order on a set A prove that R^{-1} is also a partial order on A.

$(9 \times 2 = 18 \text{ marks })$

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III.   Answer any six questions.

25. Show that the set P×P is countably infinite where P denote the set of positive integers.

26. Find a formula for the inverse of $g(x) = \dfrac{2x-3}{5x-7}$

27. Find a value of $\delta > 0$ such that $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$ where $f(x) = \sqrt{19-x}$, L=3, $x_0 = 10$ and $\varepsilon = 1$.

28. Find the domain and range of $f(x) = \dfrac{1}{\sqrt{4-x^2}}$

29. Define a partial ordering. Verify the relation " a divides b" is a partial ordering on the set of positive integers .

30. Let n denote a positive integer. Suppose a function L is defined as follows

$$L(n) = \begin{cases} 0, & if\, n = 1 \\ L\left(\left[\dfrac{n}{2}\right]\right) + 1, & if\, n > 1 \end{cases}$$ Where [x] denotes the integer function of x. Find L(25).

31. Determine whether $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology.

32. Let S={1,2,....20}. Let $\equiv$ be the equivalence relation on S defined by congruence modulo 7. Then find the quotient set $S/\equiv$

33. Sketch the relation $y \le x^2$ in the plane $\mathbb{R}^2$.

(6 × 5 = 30 Marks)

IV. Answer any two questions.

34. a) Graph the parabola $y = x^2 - 2x - 3$. Label the vertex, axis and intercepts.

b) Find the center and radius of the circle $x^2 + y^2 + 4x - 6y - 3 = 0$.

35. a) Show that $f(x) = \dfrac{x^2 + x - 6}{x^2 - 4}$ is not continuous at x = 2, but has a continuous extension to x=2. Find that extension.

b) For what values of a, $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \ge 3 \end{cases}$ is continuous at every x.

36. Let A be the set of integers and let $\sim$ be a relation on $A \times A$ defined by $(a,b) \sim (c,d)$ iff $a + d = b + c$.
(a) Prove that $\sim$ is an equivalence relation
(b) If $A = \{1, 2, ..., 9, 10\}$, find the equivalence class of $(2,5)$.

(2 × 10 = 20 Marks)

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