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Reg. No.....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2014

(U.G.—CCSS)

Mathematics (Complementary Course)

MM 4C 04—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

Part I

Answer **all** *questions.*
Each question carries ¼ weightage.

1. Apply the operator $D^2 + 3D$ on $e^{-x} + e^{2x}$.
2. Find the general solution of $y'' + y = 0$.
3. Verify whether linearly independent : e^{3x} and xe^{3x}.
4. Find the Laplace Transform of e^{a-bt}.
5. Find the inverse Laplace transform of $\frac{1}{s^3}$.
6. Define the unit step function $u(t-a)$.
7. Give an example of a function without a fundamental period.
8. Define an even function.
9. What is the *one* dimensional wave equation ?
10. Give the general form of an IVP of 1st order.
11. What is the Lipschitz condition ?
12. Find an upper bound for error in approximating using Trapezoidal rule, $n = 4$ for $\int_1^2 x^2\,dx$.

(12 × ¼ = 3 weightage)

Turn over

Part II

Answer **all** *questions.*
Each question carries 1 *weightage :*

13. Solve the IVP $9y'' + 6y' + y = 0$, $y(0) = 4$, $y'(0) = \frac{-13}{3}$.

14. Solve $x^2 y'' - 3xy' + 4y = 0$.

15. Solve $y'' + 4y = \sin 3x$.

16. Find the Laplace transform of $t \cos wt$.

17. Find the inverse Laplace transform of $\frac{1}{(s-7)^9}$.

18. Are the following functions even, odd or neither.

(i) $f(x) = x^2$. (ii) $f(x) = e^{-|x|}$.

(iii) $\sin x + \cos x$. (iv) $x|x|$.

19. Check whether $u = e^x \sin y$ is a solution of the Laplace equation.

20. Find $y_2(x)$ by Picard's iteration for the IVP $y' = 1 + y^2$, $y(0) = 0$.

21. Compute y_3 by Euler's method with $h = 0.2$ for the IVP $y' = x + y$, $y(0) = 0$.

(9 × 1 = 9 weigh

Part III

Answer any **five** *questions.*
Each question carries 2 *weightage.*

22. Solve $y'' + y = \sec x$ by the method of variation of parameters.

23. Using convolution, find the inverse Laplace transform of $\frac{1}{s^2(s-1)}$.

24. Using Trapezoidal rule, evaluate $\int_0^1 e^{-x^2}\, dx$ with $n = 10$.

25. Solve $u_{xy} = -u_x$.

26. Find the Fourier cosine series of $f(x) = \pi - x, 0 < x < \pi$.

27. Use Simpson's Rule with $n = 4$ find $\int_1^2 x\,dx$.

28. Find the inverse Laplace transform of : $\ln\left(1 + \frac{9}{s^2}\right)$.

(5 × 2 = 10 weightage)

Part IV

Answer any **two** *questions.*
Each question carries 4 *weightage :*

29. Using Laplace transforms, solve the IVP $y'' + 6y' + 8y = e^{-3t} - e^{-5t}$, $y(0) = y'(0) = 0$.

30. Find the Fourier series of :

$$f(x) = \begin{cases} k, & -\pi/2 < x < \pi/2 \\ o, & \pi/2 < x < 3\pi/2 \end{cases}$$

Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}$.

31. Solve $y' = x + y, y(0) = 0$ by the Runge-Kutta method with $h = 0.2$ in five steps.

(2 × 4 = 8 weightage)