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C 62042 (Pages : 3) Name.....

Reg. No.....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2014

(U.G.—CCSS)

Core Course—Mathematics

MM 4B 04—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours Maximum : 30 Weightage

I. Objective Type Questions (Answer *all* questions) :

1 Write the parametric equations of the circle $x^2 + y^2 = 1$.

2 Examine whether $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$ represents a parabola ellipse or hyperbola.

3 Find the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.

4 Find the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

5 Find the Taylor polynomial of order zero generated by $f(x) = \sin x$ at $a = \frac{\pi}{4}$.

6 Define absolute convergence.

7 Define an alternating series.

8 Examine whether $\sum_{n=1}^{\infty} (-1)^{n+1}$ converges or diverges.

9 Evaluate $\frac{d}{dt}\left(\tan h \sqrt{1+t^2}\right)$.

10 Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$.

11 Prove that $e^{x + ln2} = 2e^x$.

12 Evaluate $\int_0^{\pi/6} \tan 2x \, dx$.

(12 × ¼ = 3 weightage)

Turn over

II. Short answer type questions. Answer all *nine* questions.

13 Evaluate $\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta}{3+2\sin\theta}\,d\theta$.

14 Find k if $e = 10$.

15 Evaluate $\int \frac{\log_2 x}{x}\,dx$.

16 Evaluate $\lim_{x\to 0} \frac{x-\sin x}{x^3}$.

17 Show that $\ln x$ grows slower than x as $x \to \infty$.

18 Evaluate $\int_0^{\ln 2} 4e^x \sin h\,x\,dx$.

19 Examine whether the series

$$5+\frac{2}{3}+1+\frac{1}{7}+\frac{1}{2}+\frac{1}{3!}+\frac{1}{4!}+\ldots\ldots+\frac{1}{k!}+\ldots\ldots \text{ converges.}$$

20 For what values of x do the series $\sum_{n=0}^{\infty} n!\,x^n$ converges.

21 Find $\frac{dy}{dx}$ if $y = x^x$, $x > 0$.

(9 × 1 = 9 weigh

III. Short essay questions. Answer any *five* questions.

22 Find the length of the cardioid $r = 1 - \cos\theta$.

23 Find the directrix of the parabola $r = \frac{25}{10+10\cos\theta}$.

24 Graph the curve $r = 1 - \cos\theta$.

25 The co-ordinate axes are to be rotated through an angle α to produce an equation for the curve $2x^2 + \sqrt{3}\,xy + y^2 - 10 = 0$ and has no cross product term. Find α and the new equation. Identify the curve.

26 Find $\lim_{x \to \infty} x^{1/x}$.

27 Using integral test show that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \ldots\ldots + \frac{1}{n^p} + \ldots\ldots$ converges if $p > 1$ and diverges if $p \leq 1$.

28 Find the Maclaurin series for $f(x) = \sin 3x$.

(5 × 2 = 10 weightage)

IV. Essay questions. Answer any *two* questions :

29 Solve the initial value problem $e^y \frac{dy}{dx} = 2x,\ x > \sqrt{3},\ y(2) = 0$.

30 Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

31 Show that the Maclaurin series for $\cos x$ converges to $\cos x$ for every value of x.

(2 × 4 = 8 weightage)