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Name.....................................

Reg. No.....................................

THIRD SEMESTER B.Sc. DEGREE EXAMINATION NOVEMBER 2014

(U.G.–CCSS)

Complementary Course

MM 3C 03—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

Section A

Answer **all** *questions.*

Each question carries ¼ weightage.

1. Show with an example that scalar product of vectors is commutative.
2. Find the acceleration of a particle with position vector $\vec{r}(t) = [\sin t, 0, 0]$.
3. If $f = x^2 + y^2 + z^2$, find grad f.
4. What is the Cartesian form of $\vec{r}(u,v) = [ua\cos v, ub\sin v, u]$?
5. If $\vec{F}$ = grad f, then curl $\vec{F}$ = ————.
6. Find the unit vector normal to the surface $x^2 + y^2 + z^2 = 9$.
7. Verify that $y = e^x + ax^2 + bx + c$ is a solution $y''' = e^x$.
8. Solve $y' = -2xy$.
9. Test for exactness : $-\frac{y}{x^2}dx + \frac{dy}{x} = 0$.
10. Define rank of a matrix.
11. Is the matrix $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ singular *or* non-singular ?
12. State Cayley Hamilton theorem.

(12 × ¼ = 3 weightage)

Turn over

Section B

Answer **all** *questions.*
Each question carries 1 *weightage.*

13. Find the angle between $x + y + z = 1$ and $x + 2y + 3z = 6$.

14. Find a parametric representation of the straight line through (4, 2, 0) in the direction of [1, 1, 0].

15. Find the length of the semi cubical parabola $\vec{r}(t) = \left[t, t^{3/2}, 0\right]$ from (0, 0, 0) to (4, 8, 0).

16. Evaluate $\int_C \vec{F} \circ d\vec{r}, \vec{F}(\vec{r}) = [z, x, y], C : \vec{r}(t) = [\cos t, \sin t, 3t]$ and $0 \le t \le 2\pi$.

17. Use Green's theorem to find the area enclosed by the circle $x^2 + y^2 = a^2$.

18. Solve the initial value problem $2\sin 2x \sinh y dx - \cos 2x \cosh y dy = 0, y(0) = 1$.

19. Find an integrating factor : $2\cosh x \cos y dx = \sinh x \sin y dy$.

20. Find the rank of $\begin{bmatrix} 1 & 3 & 6 \\ 2 & 6 & 12 \end{bmatrix}$.

21. Find the eigen values of $\begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$.

(9 × 1 = 9 weightage)

Section C

Answer any **five** *questions.*
Each question carries 2 *weightage.*

22. (i) Find the potential function of $[yz, xz, xy]$.

    (ii) Test whether irrotational : $\vec{v} = \left[2y^2, 0, 0\right]$.

23. Test for path independence and if independent, integrate from :
    $(0,0,0)$ to $(a,b,c) : \cos(x + yz)[dx + zdy + ydz]$.

24. Evaluate $\iint_S \vec{F} \circ \vec{n}\, dA$ using Gauss divergence theorem :

    $\vec{F} = \left[x^3, y^3, z^3\right]$, S is the surface of the sphere $x^2 + y^2 + z^2 = 4$.

5. Solve $y' + y\sin x = e^{\cos x}$.

6. Solve using the transformation $\frac{y}{x} = v : 2xyy' = y^2 - x^2$.

7. Find the rank by reducing to normal form : $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$.

8. Using Cayley Hamilton theorem. Find the inverse of : $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.

(5 × 2 = 10 weightage)

Section D

Answer any* two *questions.

Each question carries 4 weightage.

9. State Stokes' theorem and verify it for $\vec{F} = \left[y^2, z^2, x^2\right]$. S being the portion of the paraboloid $x^2 + y^2 = z, y \geq 0, z \leq 1$.

. (i) Solve $y' + 2y = y^2$.

(ii) Find the Orthogonal trajectories of $y = ce^{-x}$.

1. Find the eigen values and eigen vectors of :

$$A = \begin{bmatrix} -2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 5 \end{bmatrix}.$$

(2 × 4 = 8 weightage)