16U212 (Pages: 2) Name ………………

Reg No………………

SECOND SEMESTER B. Sc. DEGREE EXAMINATION, MAY-2017

(Regular/Supplementary/Improvement)

(CUCBCSS – UG)

CC15U MAT2 B02- CALCULUS

(Core Course: Mathematics)

(2015 Admission Onwards)

Time: 3hr Max: 80 Marks

Part I. Objective type questions

Answer *all* questions (12 × 1= 12 marks)

1. Find the absolute minimum values of $g(t) = 8t - t^4$ on $[-2,1]$.
2. Prove that functions with same derivative differ by a constant.
3. Find the critical points of the function $h(x) = cosx$.
4. Find the linearization of $f(x) = cosx$ at $x = \frac{\pi}{2}$.
5. Evaluate the sum $\sum_{k=1}^{7} k(3k+5)$.
6. $\int_a^b f(x)dx + \int_b^c f(x)dx =$ -------------.
7. Show that the value of $\int_0^1 \sqrt{1+cosx}\,dx$ cannot possibly be 2.
8. State the Fundamental Theorem of Calculus.
9. If g' is continuous on the interval $[a,b]$ and f is continuous on the range of g, then $\int_a^b f(g(x)).g'(x)dx =$ ----------
10. Volume of the solid generated by revolving about the x- axis the region between the x-axis and the graph of the continuous function $y = R(x), a \le x \le b$, is ---------
11. The turning effect of a force about the origin is called
12. If W is the work done by a variable force $F(x)$ directed along the x- axis from $x = a$ to $x = b$, then W = ------.

Part II. Short answer type questions

Answer *any 9* questions (9 × 2 = 18 marks)

13. Distinguish between absolute and local extrema.
14. Let a function f be continuous on $[a,b]$ and differentiable on (a,b). If $f'(x) > 0$ at each point $x \in (a,b)$ then prove that f is increasing on $[a,b]$.
15. Find the local extreme values for the function $g(x) = -x^3 + 12x + 5,\ -3 \le x \le 3$. Where does the function assume these values?
16. Evaluate the following limit: $\lim_{x\to\infty} \frac{11x+2}{3x^3+2}$
17. What is the smallest perimeter possible for a rectangle whose area is $16cm^2$?
18. About how accurately should we measure the radius r of a sphere to calculate the surface area within 1% of its true value?
19. Using an area, evaluate $\int_0^b xdx\,, 0 < b$.

20. Evaluate $\int_{-\sqrt{7}}^{0} x(x^2+1)^{1/3}dx$.
21. Find the area of the region between the curve $y = 3x - x^2, 0 \le x \le 4$ and the x- axis.
22. Let f be continuous on $[-a, a]$. If f is odd, then prove that $\int_{-a}^{a} f(x)dx = 0$.
23. Find the area of the region enclosed by the line $y = 2$ and the curve $y = x^2 - 2$.
24. A cone $3m$ high has a base radius $3m$. The cross section of the pyramid perpendicular to the altitude x meters down from the vertex is a circle of radius x meters. Find the volume of the cone.

Part III. Short essay or paragraph questions

Answer *any 6* questions (6 × 5 = 30 marks)

25. State Rolle 's Theorem. Verify it for the function $f(x) = x^2 - 3x + 2$ on $[1, 2]$.
26. A rectangle is to be inscribed in a semi circle of radius 2. What is the largest area the rectangle can have and what are its dimensions?
27. Using limits of Riemann sums, establish the equation $\int_a^b x^2dx = \frac{b^3}{3} - \frac{a^3}{3}, a < b$
28. Find the area of the region enclosed by the curves $y = sec^2x$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{4}$.
29. Find the volume of the solid generated by the revolution the region between the y-axis and the curve $= \frac{2}{y}$, $1 \le y \le 4$, about the y- axis.
30. Find the length of the curve $y = logsec\, x$ extending from the origin to the point of intersection with the line $x = \frac{\pi}{3}$.
31. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$, about the x-axis.
32. Find the centre of mass of a 2m long rod of non-constant density $\delta(x) = \begin{cases} 2 - x, 0 \le x \le 1 \\ x, \quad 1 \le x \le 2 \end{cases}$
33. How much work does it take to pump the water from a full upright circular cylindrical tank of radius $5m$ and height $10m$ to a level of $4m$ above the top of the tank?

Part IV. Essay questions

Answer *any 2* questions (2 × 10 = 20 marks)

34. If f has a local maximum value at an interior point c of its domain and if f' is defined at c, then prove that $f'(c) = 0$. Can you find c satisfying this theorem for the function $f(x) = |x|$ on the interval $[-2, 2]$.
35. Graph the function $y = x^{5/3} - 5x^{2/3}$.
36. Prove that the length s of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ measured from $(0, a)$ to the point (x, y) is given by $s = \frac{3}{2}\sqrt[3]{ax^2}$. Also find the entire length. Find the area of the surface generated by revolving an arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}, 0 \le x \le a$, about the x-axis

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