D 70941 (Pages : 3) Name..........................34.....

Reg. No.........................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014

(UG–CCSS)

Core Course—Mathematics

MM 5B 06—ABSTRACT ALGEBRA

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*

1. Is the usual multiplication a binary operation on the set $H = \left\{ n^2 / n \in Z^+ \right\}$.

2. Find the sum of 21 and 34 modulo 45.

3. A group homomorphism is one to one if and only if Kerϕ = ——————.

4. Describe Klein 4-group.

5. Find the quotient and remainder when –38 is divided by 7.

6. Find the inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}$.

7. Define orbits of a permutation σ of a set A.

8. Find the index of the subgroup $H = \{0, 3\}$ in the group z_6.

9. Determine whether the map $\phi : (R, +) \rightarrow (Z, +)$ is given by $\phi(x)$ = greatest integer less than or equal to x is a homomorphism.

10. Give an example of a ring with unit element.

11. Define basis of a vector space.

12. Express $(3, -2, 5)$ as a linear combination of $(2, 0, 0), (0, 2, 0)$ and $(0, 0, 2)$.

(12 × ¼ = 3 weightage)

Part B

Answer **all** *questions.*

13. Find the sum of 23 and 31 modulo 45.

14. Show that the binary structures $(Q, +)$ and $(Z_r, +)$ under usual addition are not isomorphic.

Turn over

15. Show that set of all real numbers other than 1 fdm a group under the operation $a*b = a+b -$

16. Find the quotient and remainder when -38 is divided by 9 according to division algorithm,.

17. Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 2 & 6 & 1 & 3 \end{pmatrix}$ as a product of transpositions.

18. Find the cyclic subgroup generated by 8 in the group Z_{12}.

19. Prove that every group of prime order is cyclic.

20. Find the units in the ring Z_5.

21. If U and W be two subspaces of a vector space V, then prove that $U \cap V$ is also a subspace of

$(9 \times 1 = 9$ weight a

Part C

Answer any **five** *questions.*

22. Let G be a group then prove the following :

(a) $\left(a^{-1}\right)^{-1} = a$ for all $a \in G$.

(b) $(a*b)^{-1} = b^{-1}*a^{-1}$ for all $a, b \in G$.

23. Prove that a subset H of a group G is a subgroup of G if and only if the following conditions satisfied

(a) H is closed under the operation in G.

(b) The identity e of G is in H.

(c) For all $a \in H$ it is true that $a^{-1} \in H$.

24. Show that subgroup of a cyclic group is cyclic.

25. Every permutation σ of a finite set is a product disjoint cycles. Prove this theorem.

26. Find all the subgroups of Z_{18} and draw the lattice diagram.

27. If p is prime, show that Zp has no divisors of 0.

28. Find K such that $S = \{(2, -1, 3), (3, 4, -1), (K, 2, 1)\}$ is L.I.

$(5 \times 2 = 10$ weight a

Part D

Answer any two questions.

29. Prove that every group is isomorphic to a group of permutations.

30. Let ϕ be a homomorphism of a group G into a group G' then prove the following

    (a) If $a \in G$, then $\phi(a^{-1}) = \phi(a)^{-1}$.

    (b) If H is a subgroup of G, then $\phi[H]$ is a subgroup of G'.

31. Let $S_1 = \{(1, 2, 3), (0, 1, 2), (3, 2, 1)\}$ and $S_2 = \{(1, -2, 3), (-1, 1, -2), (1, -3, 4)\}$ of V_3 determine the basis and dimension of $[s_1] + [s_2]$.

(2 × 4 = 8 weightage)