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Cite as: AIP Conference Proceedings **2162**, 020069 (2019); https://doi.org/10.1063/1.5130279
Published Online: 29 October 2019

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Abstract. This paper implements Nonstandard Finite Difference Time Domain (NS-FDTD) method to analyses the scattering effect of electromagnetic wave by an artificial plasma cylinder. This NS-FDTD which requires less iteration time for convergence has proved to be around 10,000 times more accurate than the standard FDTD. For a chosen problem, the ratio between wave length and grid space in NS-FDTD need to be only 8 whereas for the standard algorithm it has to be 1140 for the same qualitative result. In this paper, NS-FDTD algorithm which is already successfully implemented for lossless and low loss medium is extended to high loss medium. An artificial negative permittivity metamaterial medium of cylindrical profile is designed using periodic arrangement of thin conducting wires and NS-FDTD is used to study the scattering properties of the plasma structure. For simplicity, interacting field quantities are realized in terms of propagation equation. Converging points of stability function are carefully selected by choosing appropriate value of conductivity. Plane wave of frequency less than the plasma frequency of the artificial plasma cylinder is used for the study. The result obtained is compared with standard FDTD which proves the obvious advantage of using this novel algorithm.

INTRODUCTION

Artificial plasma, exhibiting negative permittivity is one of the constituents of metamaterial medium [1]. Owing to the proposed applications of these materials, intense research both in experiment as well as in computation domain is happening to choke out novel methods and technologies to meet the state of the art. Commonly seen methods to simulate the electromagnetic wave propagation through artificial plasma medium are Finite Element Method (FEM), Finite Difference Time Domain (FDTD) method etc. [2, 3, 4]. Since these methods takes greater computational time, we introduce a modified version of FDTD called nonstandard FDTD (NS-FDTD), which is highly accurate and take less computational time, to study scattering effect in artificial plasma medium [5, 6]. NS-FDTD is first introduced by Cole and it is proved that this algorithm is 10,000 times more accurate than standard FDTD [5]. Comparing accuracy of NS- FDTD and standard FDTD, the accuracy obtained using standard FDTD for the ratio between wave length λ and grid space h, λ/h = 1140 can be achieved at λ/h = 8 using NS-FDTD. NS-FDTD is already successfully implemented for lossless cases and it is also used to demonstrate the scattering of electromagnetic waves by dielectric cylinder [7, 8]. NS-FDTD is also modeled for lossy medium and applied for low loss medium [9]. Recently, a major step in the implementation of NS-FDTD is reported when it is successfully applied for the case of conducting wire medium which resulted in its use in a metamaterial medium for the first time [6]. In this paper, we have applied this powerful algorithm to study scattering effect of artificial plasma

Proceedings of the International Conference on Advanced Materials
AIP Conf. Proc. 2162, 020069-1–020069-5; https://doi.org/10.1063/1.5130279
Published by AIP Publishing. 978-0-7354-1907-0/$30.00

cylinder (negative permittivity metamaterial medium) by implementing this algorithm for high loss cases. Artificial plasma cylinder is a negative permittivity material which is made by the periodic arrangement of thin conducting wires. To study the scattering effects, a sine wave of frequency less than the plasma frequency is allowed to interact with the plasma cylinder. The results are compared with standard FDTD algorithm in order to bring out the obvious advantages of NS - FDTD. Interaction of plane wave of same frequency with the plasma cylinder is also studied using NS-FDTD.

NS-FDTD METHOD FOR LOSSY MEDIUM

In order to model the negative permittivity artificial plasma cylinder using periodic thin wires, we use the absorbing wave equation for the electric field E_z, which is given by [9]

$$(\partial_{tt} - v^2\nabla^2 + 2\alpha\partial_t)\, E_z(\mathbf{x}, t) = 0 \tag{1}$$

where $\alpha = \sigma/2\varepsilon_0$ is the absorption factor with σ as conductivity, v is the velocity parameter and $\mathbf{x} \equiv (x, y, z)$ denotes vector position.

A real analytic solution to Eq. 1 in 2 dimensions is

$$E_z(\mathbf{x}, t) = e^{-\alpha t}\sin(\omega t)\,\sin(\mathbf{k}.\mathbf{x}) \tag{2}$$

where the propagating vector, $\mathbf{k} = (k_x, k_y)$ and

$$\omega = \sqrt{\omega_0^2 - \alpha^2} \tag{3}$$

$$\omega_0 = kv \tag{4}$$

The standard finite difference (FD) approximation to $\nabla^2 E_z(\mathbf{x}, t)$ is

$$\nabla^2 E_z(\mathbf{x}, t) = \frac{D_1^2}{h^2} E_z(\mathbf{x}, t) \tag{5}$$

where

$$D_1^2 = d_{xx} + d_{yy} \tag{6}$$

Applying second order central finite difference approximation to Eq. 1 gives us

$$\left(\partial_{tt} - \frac{v^2}{h^2} D_1^2 + 2\alpha\partial_t\right) E_z(\mathbf{x}, \mathrm{t}) = 0 \tag{7}$$

To implement NSFD approximation, $\nabla^2 E_z(\mathbf{x}, t)$ can be written as [9]

$$\nabla^2 Ez(\mathbf{x}, t) = \frac{D_0^2}{s(ik,h)^2} E_z(\mathbf{x}, \mathrm{t}) \tag{8}$$

where $s(ik, h)$ is the correction function in the case of NS-FDTD for the grid space h and D_0^2 is the weighted superposition of two independent difference operators (D_1^2, D_2^2) of the form

$$D_0^2 = \gamma\, D_1^2 + (1-\gamma) D_2^2 \tag{9}$$

where

$$2D_2^2 E_z(x, y) = E_z(x+h, y+h) + E_z(x+h, y-h) + E_z(x-h, y+h) + E_z(x-h, y-h) - 4E_z(x, y) \tag{10}$$

and γ, the weighing function is

$$\gamma = \frac{2}{3} - \frac{1}{90}(kh)^2 - \frac{1}{15120}(kh)^4\,(11 - 5\sqrt{2}) - \cdots \tag{11}$$

Hence Eq. 1 become

$$(\partial_{tt} - \frac{v^2}{h^2} D_0^2 + 2\alpha\partial_t)E_z(\mathbf{x}, \mathrm{t}) = 0 \tag{12}$$

Solving Eq. 12, we get

$$E_z(\mathbf{x}, t + \Delta t) = E_z(\mathbf{x}, t) + \left(\frac{1-a}{1+a}\right)[E_z(\mathbf{x}, t) - E_z(\mathbf{x}, t - \Delta t)] + (\frac{u^2}{1+a})\, D_0^2 E_z(\mathbf{x}, t) \tag{13}$$

where u^2 and a are given by

$$u^2 = \frac{1}{sin^2\left(\frac{kh}{2}\right)}[sin^2\left(\frac{\omega\Delta t}{2}\right) cosh^2\left(\frac{\alpha\Delta t}{2}\right) - sinh^2\left(\frac{\alpha\Delta t}{2}\right) cos^2\left(\frac{\omega\Delta t}{2}\right) + \frac{1}{2}\tanh(\alpha\Delta t)\sinh(\alpha\Delta t)\cos(\omega\Delta t)] \tag{14}$$

$$a = \tanh(\alpha\Delta t) \tag{15}$$

In order to implement NS-FDTD for a conducting medium, we need to carefully select the values of σ in relation with the discretization parameter λ for the proper convergence of the stability function u^2.

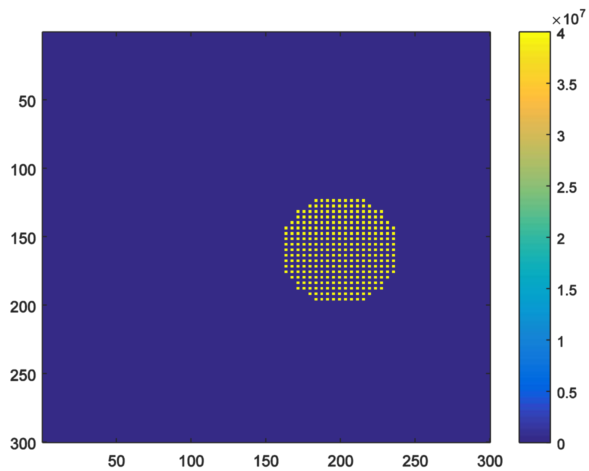


FIGURE 1: Positioning of the negative permittivity plasma cylinder in the computational domain

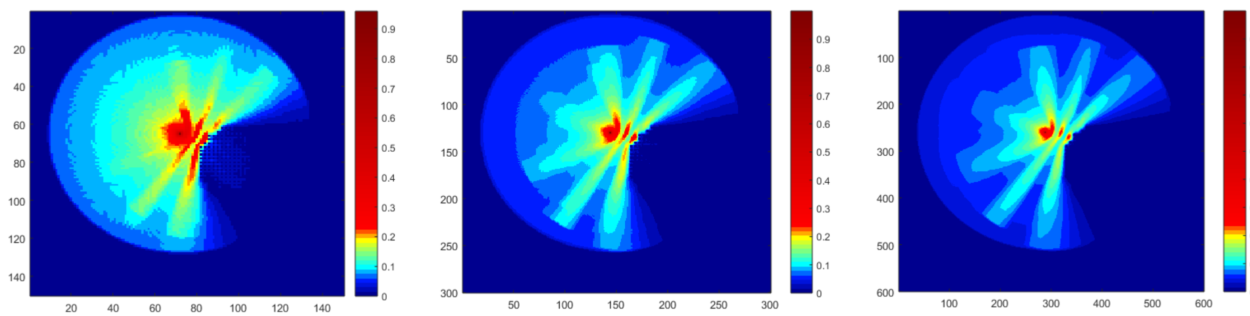
DESIGNING OF PLASMA CYLINDER IN THE COMPUTATIONAL DOMAIN

The artificial plasma cylinder of radius 2.5λ constructed in the computational domain is shown in Fig. 1 with its center at a distance of 8.75λ from the bottom and 12.5λ from the left end. The plasma cylinder is modeled by the periodic arrangement of conducting wires of area 6.9444 x 10^{-15} m^2 with periodicity b = 1.6667x 10^{-7} m in $x - y$ plane. The plasma frequency f_p is calculated using the equation [10]

$$f_p = \frac{c}{b\sqrt{2\pi \ln(\frac{b}{r})}} \tag{16}$$

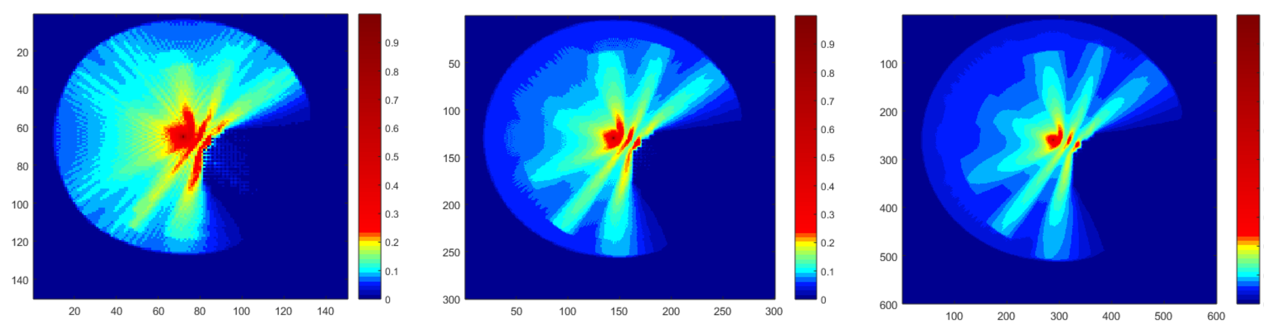
where r is the equivalent radius calculated from the cross sectional size of the conducting wire. Sine wave of frequency 4.5×10^{14} Hz used as an excitation source is placed on the left side of the plasma cylinder with 9.75λ from the bottom and 8.125λ from the left end. The domain selected for simulation is of dimensions 18λ × 18λ cells and each grid is of size λ/N. The time step Δt used for the simulation is set as 0.8h/c.

RESULTS AND DISCUSSION



(a) (b) (c)

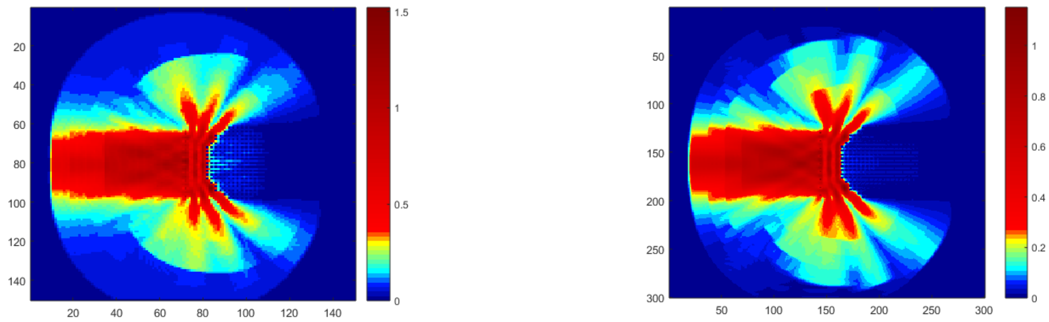
FIGURE 2: Electric field distribution near the artificial plasma cylinder when it interacts with a sine wave using NS-FDTD (a) N=8, (b) N=16 and (c) N=32



(a) (b) (c)

FIGURE 3: Electric field distribution near the artificial plasma cylinder when it interacts with a sine wave using standard FDTD (a) N=8, (b) N=16 and (c) N=32

In order to model conducting wire in computational domain using NS-FDTD, we have to select its conductivity value (σ) carefully for successful simulation. In this case, value of σ selected is nearly equal to 4 x 10^7 S/m. Scattering of sine pulse by artificial plasma cylinder is analyzed using NS-FDTD and standard FDTD for different values of discretization parameter *N*. Simulation results using NS-FDTD for N=8, 16, 32 are shown in Figs. 2 (a), (b) and (c). Corresponding results using standard FDTD are shown in Figs. 3 (a), (b) and (c) for same N values. From the comparison, we can understand that NS-FDTD gives good scattering profile even for low values of N. Here the number of iterations needed is also less. Standard FDTD requires very large value of N for getting the same accuracy which NS-FDTD can achieve in small value of N. The cylinder is also allowed to interact with plane wave of same frequency and the field distributions of the propagating waves are shown in Figs. 4 (a) and (b). From these figures, it is quite clear that NS-FDTD algorithm gives high accuracy results for low discretization values.



(a) (b)

FIGURE 4: Scattered electric field distribution due to the artificial plasma cylinder when it interacts with a plane wave using NS-FDTD (a) N=8, (b) N=16

CONCLUSION

Scattering of electromagnetic wave by artificial plasma cylinder is analyzed using NS-FDTD and standard FDTD methods. Sine wave of frequency less than the plasma frequency is used for analysis. From the profile obtained using NS-FDTD and standard FDTD, we can understand NS-FDTD method is highly accurate and less time consuming. In this paper, we have successfully demonstrated that the use of NS-FDTD algorithm in negative permittivity artificial plasma medium in the form of the cylinder, where by all the stated advantages of this powerful algorithm is made available for metamaterial structures.

ACKNOWLEDGMENTS

Jovia Jose gratefully acknowledges UGC of India for the financial support under Faculty Development Program (FDP). Sikha K. Simon gratefully acknowledges UGC of India for the financial support under MANF-JRF. Sreedevi P. Chakyar gratefully acknowledges UGC of India for the financial support under UGC- JRF.

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