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FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL/MAY 2015

(U.G.—CCSS)

Core Course—Mathematics

MM 4B 04—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all* questions :—

1 Find y if $ln\ y = 3t + 5$.

2 Evaluate $\int\limits_{0}^{\pi/2} e^{\sin x}\ \cos x\ dx$.

3 Evaluate $\dfrac{d}{dx}\ ln_{10}(3x + 1)$.

4 Evaluate $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + x} - 1}{x}$.

5 Integrate $\coth 5x$.

6 Examine whether $\sum\limits_{n=1}^{\infty} \dfrac{n+1}{n}$ converges or diverges.

7 Define an alternating series.

8 Find the Taylor series for $f(x) = e^x$ at $x = 0$.

9 Find the Taylor polynomial of order 0 generated by $f(x) = \dfrac{1}{x}$, $a = 2$.

10 Find the focus of the parabola $y^2 = 10x$.

11 Find the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.

12 Write the parametric equation of the circle $x^2 + y^2 = a^2$.

(12 × ¼ = 3 weightage)

Turn over

II. Answer all *nine* questions :—

13 Evaluate $\int\limits_{-\pi/2}^{\pi/2} \dfrac{4\cos\theta}{3+2\sin\theta}\,d\theta$.

14 Integrate $2^{\sin x}\cos x$ with respect to x.

15 Evaluate $\lim\limits_{x\to 0}\dfrac{x-\sin x}{x^3}$.

16 Show that $ln\,x$ grows slower than x as $x\to\infty$.

17 Examine whether $x^2+xy+y^2-1=0$ represents a parabola, ellipse or hyperbola.

18 Find the vertices of the ellipse $\dfrac{x^2}{16}+\dfrac{y^2}{9}=1$.

19 Find the Taylor series for $f(x)=\cos x$ at $x=0$.

20 For what values of x do the series $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\dfrac{x^n}{n}$ converges.

21 Examine whether the series

$$5+\frac{2}{3}+1+\frac{1}{7}+\frac{1}{2}+\frac{1}{3!}+\frac{1}{4!}+\dots\dots+\frac{1}{k!}+\dots\dots \text{ converges.}$$

(9 × 1 = 9 weighta

III. Answer any *five* questions :—

22 Find the directrix of the parabola $r=\dfrac{25}{10+10\cos\theta}$.

23 Graph the curve $r^2=4\cos\theta$.

24 Find the tangent to the right-hand hyperbola branch $x=\sec t,\ y=\tan t,\ -\pi/2<t<\dfrac{\pi}{2}$ at

point $\left(\sqrt{2},1\right)$ where $t=\dfrac{\pi}{4}$.

25 Show that the Maclaurin's series for $\sin x$ converges to $\sin x$ for all x.

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26 Investigate the converges of the series $\sum\limits_{n=1}^{\infty} \dfrac{(2n)!}{n!\,n!}$.

27 Evaluate $\int\limits_{0}^{1} \sinh^2 x\, dx$.

28 Show that $\lim\limits_{x \to 0^+} (1+x)^{\frac{1}{x}} = e$.

(5 × 2 = 10 weightage)

IV. Answer any *two* questions :

29 Find the sum of the series $\sum\limits_{n=1}^{\infty} \dfrac{1}{n(n+1)}$.

30 Multiply the geometric series $\sum\limits_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots + x^n + \dots \dfrac{1}{1-x}$, for $|x| < 1$, by itself to

get a power series for $\dfrac{1}{(1-x)^2}$, for $|x| < 1$.

31 Find the length of the astroid $x = \cos^3 t$, $y = \sin^3 t$, $0 \le t \le 2\pi$.

(2 × 4 = 8 weightage)