90906 (Pages : 3) Name..............................45

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FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(UG—CCSS)

Core Course—Mathematics

MM 5B 06—ABSTRACT ALGEBRA

Time : Three Hours Maximum : 30 Weightage

Part A

Questions from 1 to 12 *are* compulsory.

Each has weight ¼.

1. On Q define '*' by $a * b = a/b$. Is * a binary operation on Q ?
2. State True *or* False :
   "Any two groups of three elements are isomorphic".
3. Write a non-trivial proper Subgroup of Klein 4-group.
4. Find the subgroup of Z_{12} generated by 3.
5. Give an example of a cyclic group having only one generator.
6. What is the order of the cycle $(1, 4, 5, 7)$ in the group S_8 ?
7. State True *or* False :
   "All subgroups of abelian groups are normal".
8. The index of the alternating group A_n in the symmetric group S_n, $n > 1$ is ————.
9. Compute the product (12) (16) in Z_{24}.
10. Give the number of divisions of zero in Z_4.
11. Is the set $\{(1,1), (0,0)\}$ Linearly Independent in $\mathbb{R}^2$?
12. Find an element in the span of $\{1-x, x-x^2, 1+x^2\}$.

(12 × ¼ = 3 weightage)

Part B (Short Answer Type Questions)

Answer **all** *questions (from 13 to 21).*

Each question has weight 1.

13. For all a, b in a Group G prove that $(a * b)^1 = b^1 * a^1$.

Turn over

14. Find the order of the subgroup of Z_4 generated by 2.

15. Determine whether the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=x^2$ is a permutation of R.

16. Define orbits of a permutation.

17. Prove that Every permutation σ of a finite set is a product of disjoint cycles.

18. Let H be a subgroup of a finite group G. Then prove that the order of H is a divisor of the o G.

19. Find all solutions of the equation $x^2+2x+3=0$ in Z_6.

20. Let R be a ring with unity. If $n\cdot 1\neq 0$ for all $n\in z^{+}$, then prove that R has characteristic 0

21. Show that the set of all continuous functions $f(x)$ on $[a,b]$ such that $f\left(\frac{a+b}{2}\right)=1$, with addition of functions and multiplication by real numbers is not a vector space.

(9 × 1 = 9 weig

Part C (Short Essay Questions)

Answer any **five** *questions (from 22 to 28).*
Each question has weight 2.

22. Show that the set of all m × n matrices with real entries under matrix addition is an abelian g

23. Find all subgroups of Z_{18} and give their subgroup diagram.

24. Define even permutation. Check whether the permutation $\sigma=\begin{pmatrix}1&2&3&4&5&6&7&8\\3&6&4&1&8&2&5&7\end{pmatrix}$ is an even permutation ?

25. Let ϕ be a homomorphism of a group G into a group G^1. Then prove the following :

    (1) If e is the identity elements in G, then $\phi(e)$ is the identity element in G^1.

    (2) If $a\in G$, then $\phi(a^{-1})=[\phi(a)]^{-1}$.

    (3) If H is a subgroup of G, then $\phi(H)$ is a subgroup of G^1.

26. Prove that the cancellation laws hold in a ring R if and only if R has no divisors of 0.

27. Prove that every field F is an integral domain. Is the converse true ? Justify your answer.

28. Let $V=p_4$ be the vector space of polynomials of degree ≤ 4 with usual addition and s multiplication and let $U=\{p(x)\in p_4/p''(1)=2p^1(1)\}$. Show that U is a subspace of P_4.

(5 × 2 = 10 weigh

Part D (Essay Questions)

Answer any **two** *questions (from 29 to 31).*
Each question has weight 4.

29. Describe two different group structures of order 4 and show that one of them is isomorphic to the group of fourth roots of unity under multiplication.

30. State and prove Cayley's theorem.

31. (a) Define dimension of a vector space.

(b) Let U and W be subspaces of R^3 defined by :

$U = \{(x_1, x_2, x_3) / x_1 + x_2 - 2x_3 = 0\}$ and $W = \{(x_1, x_2, x_3) / x_1 - 3x_2 + 2x_3 = 0\}$. Find dim U, dim W and dim $U \cap W$, by finding their bases and show that $U + W = R^3$.

(2 × 4 = 8 weightage)