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Name.....

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Reg. No.....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(CUCBCSS—UG)

Core Course—Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours

Maximum : 80 Marks

Part A

Objective type : Answer all twelve questions.

1. The natural logarithm is the function given by _____.
2. $\lim_{x \rightarrow 2} \frac{x + 2}{x^2 - 4} =$ _____.
3. The hyperbolic secant is defined as _____.
4. The sequence $\{a_n\}$ diverges to infinity if _____.
5. If $|r| < 1$, the geometric series $a + ar + \dots + ar^{n-1} + \dots$ converges to _____.
6. Suppose that $a_n > 0$ and $b_n > 0$ for $n \geq N$. If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$, then _____.
7. The Maclaurin series of $f(x) = \sin \frac{x}{2}$ is _____.
8. The first two terms in the expansion of $f(x) = e^x \cos x \frac{n!}{r!(n-r)!}$ as a Maclaurin series is _____.
9. The Taylor series expansion of $f(x)$ about a point $x = a$ is _____.
10. The eccentricity of the conic section $r = \frac{1}{1 + \cos \theta}$ is _____.
11. The centre of the circle $r = -2 \cos \theta$ is _____.
12. The standard form of the ellipse with foci $(\pm 8, 0)$ is _____.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer type)

Answer any nine questions.

13. Express the logarithm in $\left(\frac{1}{125}\right)$ in terms of $\ln 5$ and $\ln 7$.
14. Express the number e as a limit.
15. Find $\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$.
16. Determine whether $\sum_{n=1}^{\infty} \frac{n+1}{n}$ converges or diverges?
17. Determine whether the series $\sum_{n=1}^{\infty} \left(\frac{1}{1+n}\right)^n$ converge or diverge?
18. Determine whether the Alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{3/2}}$ converges or diverges?
19. Define power series representation of a function.
20. Find the power series representation of $f(x) = \frac{1}{1-x}$.
21. Find the first two terms in the expansion of $f(x) = \cos x$ at $x = 0$.
22. What is the polar equation of a conic with eccentricity e ?
23. Find the eccentricity of the ellipse $2x^2 + y^2 = 4$.
24. Sketch the circle $r = 4 \cos \theta$.

(9 × 2 = 18 marks)

Part C (Short Answer Type)

Answer any six questions.

25. Define Hyperbolic function.
26. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \dots + \frac{1}{n} + \dots$ converge or diverge?

1. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{4^n n! n!}{(2n)!}$.
2. Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$ converge or diverge?
3. Define Maclaurin series expansion of a function. Expand $f(x) = \sqrt{x+1}$ about a point $a = 0$.
4. Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x-2)^2}{10^n}$.
5. What is a parametrisation of a curve in xy plane? Give some typical parametrisation for lines, circles, parabolas, ellipses and hyperbolas.
6. Find the eccentricity and directrix of the parabola $r = \frac{25}{10 + 10 \cos \theta}$. Also sketch the parabola.
7. Find the polar equation of the conic section :
- (i) $r \cos \theta = 2, e = 2$.
- (ii) $r \cos \theta = -4, e = 1$.

(6 × 5 = 30 marks)

Part D (Essay Type)*Answer any two questions.*

1. (i) Consider the series $a_n = \frac{\frac{n}{2^n}, n \text{ odd}}{\frac{1}{2^n}, n \text{ even}}$. Does $\sum a_n$ converge?
- (ii) Does $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converge?
2. Find the Taylor series generated by $f(x) \frac{1}{x}$ at $a = 2$. Does this series converges to $\frac{1}{x}$?
3. Define eccentricity of a conic section. Classify the conic section by eccentricity. How are an ellipses shape and eccentricity related? Give examples.

(2 × 10 = 20 marks)