15U329 (Pages:3) Name: …………………….

Reg. No……………………

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS - UG)

CC15U ST3 C03 - STATISTICAL INFERENCE

Statistics - Complementary Course

(2015 Admission)

Time: Three Hours Maximum: 80 Marks

Part A

(Answer ***all*** questions. Each question carries **one** mark)

1. If $\overline{X}$ is the mean of a sample of size 25 taken from a normally distributed population $N(12, 3^2)$, then $E(\overline{X}) =$ -----------
2. The standard deviation of a statistic is known as -------------
3. If $Z \sim N(0,1)$, then Z^2 follows ---------- distribution
4. If s^2 is the variance of a sample of size n taken from $N(\mu, \sigma^2)$ then $E\left(\frac{ns^2}{\sigma^2}\right)$ is ------
5. The mode of student's 't' distribution is ---------
6. If $F \sim F(5, 5)$, then $\frac{1}{F}$ follows ------ distribution
7. If t_n is a consistent estimator of θ, then as 'n' tends to infinity $V(t_n)$ tends to -------
8. Let t_1 and t_2 be two unbiased estimators of θ. Then t_1 is more efficient than t_2 if ----------
9. A hypothesis which completely specifies parameters of the distribution followed by the population is called ------
10. The test for goodness of fit is based upon ----------- distribution.

(10 x 1 =10 marks)

Part B

(Answer ***all*** questions. Each question carries **two** marks)

11. Define sampling distribution .
12. A random sample $X_1, X_2, \ldots, X_9$ is taken from a population that follow $N(10, 6^2)$. Find $E\left(\sum_{i=1}^{9}\left(\frac{X_i - 10}{6}\right)^2\right)$
13. Define student's 't' distribution with 'n' degress of freedom and show that for n = 1 it coincides with Cauchy distribution
14. Show that sample variance s^2 is biased but consistent estimator of population variance σ^2 where the sample is drawn from a population that follow $N(\mu, \sigma^2)$.
15. State Fisher Neyman factorization theorem
16. Define significance level and power of test.
17. Give any two applications of Chi-square distribution in testing of hypothesis.

(7 x 2 = 14 Marks)

Part C

(Answer ***any three*** questions. Each question carries **four** marks)

18. Let $X_1, X_2, \dots, X_n$ be a random sample drawn from $N(\mu, \sigma^2)$. Derive the sampling distribution of $\overline{X}$ where $\overline{X}$ is the mean of the sample .
19. If χ^2 follows chi-square distribution with 'n' degrees of freedom show that $Z = \frac{\chi^2 - n}{\sqrt{2n}}$ follows N(0, 1) when $n \rightarrow \infty$.
20. Show that sample mean is a sufficient estimator of θ where θ parameter of population which has density $f(x, \theta) = \theta e^{-\theta x}, x > 0$
21. Explain the method of constructing 95% confidence interval for the proportion 'p' of possessing a characteristic in a population.
22. A single observation X is drawn from a population following the probability density function $f(x) = (1 + \theta)x^{\theta}$, $0 < x < 1$, $\theta > 0$. The C R for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$ is $x \geq 0.8$. Find size and power of the test.

(3 x 4 = 12 Marks)

Part D

(Answer ***any four*** questions. Each question carries **six** marks)

23. Derive the moment generating function of chi-square distribution and verify the additive property.
24. Show that $t = \frac{(\overline{X} - \mu)\sqrt{n-1}}{s}$ follows student's 't' distribution with (n-1) degrees of freedom where $\overline{X}$ and s are the mean and standard deviation of a sample taken from a population that follow $N(\mu, \sigma^2)$ distribution.
25. Derive the mean and mode of 'F' distribution and hence show that mean is always greater than mode.
26. Explain the method of moment estimation. Find the moment estimator of θ where θ is the parameter of a population with following distribution.

x	0	1	2	3	4	5
p(x)	θ	2θ	$\frac{\theta}{2}$	θ	$\frac{\theta}{4}$	θ

27. A sample drawn from a normally distributed population gave following values. 45, 34, 56, 42, 39, 54, 48, 40, 45, 57. Using this data test find 95% confidence interval for mean of the population.
28. Explain the method of testing the equality of two population proportions.

(4 x 6 = 24 Marks)

Part E

(Answer ***any two*** questions. Each question carries **ten** marks)

29. a) Derive the mean and variance of Chi-square distribution

    b) Show that all raw and central moments of student's 't' distribution are equal and hence its all odd central moments are zero.

30. Let Z and χ^2 be independent random variables and follow according to N(0, 1) and χ^2 distributions with 'n' degrees of freedom respectively. Derive the distribution of $t = \frac{z}{\sqrt{\chi^2/n}}$

31. a) Show that $t = \frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)^2$ is an unbiased and consistent estimator of the σ^2 when the distribution of the population is $N(\mu, \sigma^2)$

    b) Based upon a sample of size n taken from a population with probability density function $f(x, \theta) = \binom{k}{x}\theta^x(1-\theta)^{k-x}$ derive the maximum likelihood estimator of θ

32. a) Two sample of people consisting of 100 carpenters and 80 masons have average daily wages 950.34 and 945.65 cms with standard deviations 16.24 and 13.15 respectively. Examine whether the average daily wage of carpenters is greater than that of masons at 5% level of significance.

    b) Explain the method of testing goodness of fit.

(2 x 10 = 20 Marks)

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