15U306 (Pages:2) Name: …………………

Reg. No…………………..

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS - UG)

Mathematics - Core Course

CC15U MAT3 B03- CALCULUS AND ANALYTIC GEOMETRY

(2015 Admission)

Time: Three Hours   Maximum: 80 Marks

PART-A

(Objective Type)Answer **ALL** Questions

1. Evaluate $\int \frac{(lnx)^2}{x} dx$.
2. If $\sinh x = \frac{-3}{4}$, then $\cosh x = \cdots$.
3. Evaluate $\lim\limits_{x \to 1} \left(\frac{1-x}{lnx}\right)$
4. Give an example of a bounded sequence.
5. Evaluate $\lim\limits_{n \to \infty} \left(\sqrt[n]{n}\right)$
6. Find the sum of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$
7. Give an example of an alternating convergent series.
8. Write the vertex of the parabola $(x + 2)^2 = -4(y + 3)$.
9. Find the slope of the curve $x = \sin t$ and $y = \cos t$ at $t = \pi/4$.
10. Replace the Cartesian equation $x = y$ by equivalent polar equation.
11. Write the polar equation of a circle with centre at $(a, 0)$ and passing through the origin.
12. Write the standard polar equation of the ellipse.   **(12 x 1 =12 Marks)**

PART-B (Short Answer Type)

Answer **Any 9** Questions

13. Evaluate $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.
14. Prove that $\text{sech}^{-1} x = \cosh^{-1}\left(\frac{1}{x}\right)$.
15. Evaluate $\lim\limits_{x \to 0} \left(\frac{x - \sin x}{x^3}\right)$.
16. Show that the function 3^x grows faster than 2^x.
17. Evaluate $\lim\limits_{n \to \infty} \left(\frac{n!}{n^n}\right)$.
18. Find the limit of the convergent sequence $\{a_n\}$, where $a_n = \left(\frac{1}{n}\right)^{1/\ln n}$.
19. Investigate the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n^n}{n!}\right)$.
20. Find the eccentricity of the ellipse $16x^2 + 9y^2 = 144$.

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21. Graph the set of points whose polar coordinates satisfy the conditions $0 \leq \theta \leq {}^{\pi}/_4$, $r \geq 0$

22. Find the area of the Lemniscates of Bernoulli $r^2 = a^2 \cos 2\theta$.

23. Polar equation of a conic is $r = \frac{12}{3+3\sin\theta}$. Identify the conic and find its directrix.

24. Find the equation of the tangent of the curve $x = t, y = \sqrt{t}$ at $t = 1/4$.

(9 x 2 =18 Marks)

PART-C (Short Essay Type)
Answer **Any 6** Questions

25. Express $\sinh^{-1} x \ and \ \cosh^{-1} x$ in terms of logarithms.

26. Show that $\sqrt{10x+1}$ and $\sqrt{x+1}$ grow at the same rate as $x \to \infty$.

27. Find $\lim\limits_{n\to\infty} \left(1 + \frac{x}{n}\right)^n$.

28. Test the convergence of the series $2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \cdots + \frac{(n+1)x^n}{n^3} + \cdots$

29. Show that the series $\sum_{n=1}^{\infty}(-1)^n\left[\sqrt{n+1} - \sqrt{n}\right]$ converges conditionally.

30. Find the Taylor series generated by $f(x) = e^x$ at $x = 0$. Where if anywhere, does the series converge?

31. Find the centre, eccentricity, focii and diretrices of the hyperbola
$$9x^2 - 16y^2 + 18x + 32y - 151 = 0.$$

32. Find the area of the surface of the solid formed by the revolution of the cardioid $r = a(1 + \cos\theta)$ about the $x - axis$.

33. Find the length of the cardioid $r = 1 + \cos\theta$.

(6 x 5 =30 Marks)

PART-D (Essay Type)

Answer **Any 2** Questions

34. Show that the p-series $\sum_{n=1}^{\infty}\frac{1}{n^p}$, where p is a real constant, converges if $p > 1$ and diverges if $p \leq 1$.

35. Find Maclaurin series for $f(x) = \ln(1 + x)$. Also find Taylor polynomials of orders 0, 1, 2 and 3 generated by f at zero.

36. Find the length of the asteroid $x = \cos^3(t), y = \sin^3(t), 0 \leq t \leq 2\pi$.

(2 x 10 =20 Marks)

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