15U212 (Pages: 2)

Name…………………

Reg No………………

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, JUNE 2016

(CUCBCSS- UG)

(Core Course: Mathematics)

CC15U MAT2 B02- CALCULUS

(2015 Admission)

Time: 3hr **Max Marks: 80**

Part I. Objective type questions

Answer *all* questions (12 × 1= 12 marks)

1. Define absolute maximum of a function.
2. Find the intervals on which the function $f(x) = \frac{1}{x}$ is increasing.
3. Find the critical points of the function $f(x) = x^{1/3}(x-4)$.
4. Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 0$.
5. Express $\lim_{||P||\to 0} \sum_{k=1}^{n} {c_k}^2 \Delta x_k$ as a definite integral.
6. $\int_a^a f(x)dx =$ ------------
7. Use the inequality $cosx \leq (1-\frac{x^2}{2})$ to find a lower bound for $\int_0^1 cosx\, dx$.
8. State the Fundamental Theorem of Calculus.
9. If f is odd, then $\int_{-a}^{a} f(x)dx =$ ---------
10. Volume of the solid generated by revolving about the x- axis the region between the x-axis and the graph of the continuous function $y = R(x), a \leq x \leq b$, is --------
11. The turning effect of a force about the origin is called
12. What is the SI unit of work done?

Part II. Short answer type questions

Answer *any 9* questions (9 × 2 = 18 marks)

13. Find the local and absolute extrema of the function $f(x) = x^{2/3}$ on $[-2, 3]$.
14. Prove that functions with zero derivatives are constant.
15. Find the local extreme values for the function $f(x) = sin\,x, -\frac{\pi}{3} \leq x \leq \pi$. Where does the function assume these values?
16. Find asymptotes of the graph of $f(x) = \frac{x^2-3}{2x-4}$
17. Find two positive numbers whose sum is 20 and whose product is as large as possible.
18. About how accurately should we measure the radius r of a sphere to calculate the surface area within 1% of its true value?
19. Using an area, evaluate $\int_a^b x dx$, $0 < a < b$.

20. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(2 + tan\frac{\theta}{2})sec^2\frac{\theta}{2}\,d\theta$.

21. Find the area of the region between the curve $y = 4 - x^2, 0 \leq x \leq 3$ and the x- axis.

22. Let f be continuous on $[-a, a]$. If f is even, then prove that $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$.

23. Find the area between $y = sec^2x$ and $y = \sin x$ from 0 to $\pi/4$.

24. A pyramid $3m$ high has a square base that is $3m$ on a side. The cross section of th pyramid perpendicular to the altitude x meters down from the vertex is a square x meter on a side. Find the volume of the pyramid.

Part III. Short essay or paragraph questions

Answer *any 6* questions (6 × 5 = 30 marks)

25. State and prove Mean value theorem.

26. You are planning to make an open rectangular box from an $8m \times 15m$ piece of card board by cutting squares from the corners and folding up the sides. What are the dimensions of the box of largest volume that you can make in this way?

27. Using limits of Riemann sums, establish the equation $\int_{a}^{b} x\,dx = \frac{b^2}{3} - \frac{a^2}{3}, a < b$.

28. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and $y = -x$.

29. Find the volume of the solid generated by the revolution the region between the curve $y^2 = x$, and the lines $y = 1, x = 4$ about the line $y = 1$.

30. Find the length of the curve $y^2 = x^3$, extending from the origin to the point (1, 1).

31. Find the area of the surface generated by revolving the curve $y^2 = 4ax, 0 \leq x \leq a$, about the x-axis.

32. Find the center of mass of a 10m long rod of non-constant density $(x) = 1 + \frac{x}{10}$.

33. A leaky 5-lb bucket is lifted from the ground into the air by pulling in 20ft of rope at a constant speed. The rope weighs 0.08 lb/ft. the bucket starts with 16-lb of water and leaks at a constant rate. It finishes draining just as it reaches the top. How much work was spent in lifting the water, bucket and rope?

Part IV. Essay questions

Answer *any 2* questions (2 × 10 = 20 marks)

34. If f has a local maximum value at an interior point c of its domain and if f' is defined at c, then prove that $f'(c) = 0$. Can you find c satisfying this theorem for the function $f(x) = x^{2/3}$ on the interval $[-2, 2]$.

35. Graph the function $y = x^4 - 4x^3 + 10$

36. Prove that the length s of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ measured from $(0, a)$ to the point (x, y) is given by $s = \frac{3}{2}\sqrt[3]{ax^2}$. Also find the entire length. Find the area of the surface generated by revolving an arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}, 0 \leq x \leq a$, about the x-axis.

****