15U502 (Pages: 2) Name: ……………………

Reg. No…………………..

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, OCTOBER 2017

(CUCBCSS-UG)

CC15UMAT5 B06-ABSTRACT ALGEBRA

(Mathematics - Core Course)

(2015-Admission Regular)

Time: Three Hours Maximum: 120 Marks

Section A

Answer ***all*** the questions. Each question carries 1 mark.

1. Give an example of an abelian group.
2. The smallest order of a non abelian group is …….
3. Write the subgroup of Z_{12} generated by 8.
4. State True/False: f: **RR** defined by $f(x) = x^3$ is a permutation on R.
5. Order of 4 in Z_{10} is ……
6. Number of elements in the group A_3 is ………
7. Find all the orbits of where (n) = n+1.
8. Define homomorphism from a group G to another group G.
9. How many homomorphisms are there of **.**
10. Which are the units in the ring of integers.
11. Characteristic of ring Z_9 is …..
12. State True/False: Q is a field of quotients of **.**

(12 x 1 = 12 Marks)

Section B

Answer ***any ten*** questions.Each question carries 4 marks

13. Show that if there is an identity element in a binary structure, it is unique.
14. Let be defined on Q^+ by Show that is a group.
15. Show that intersection of two subgroups of a group is a subgroup.
16. Describe all the elements in the cyclic subgroup generated by of GL(2,R).
17. Show that every cyclic group is abelian.
18. Show that a finite cyclic group of order n is isomorphic to Z_n.
19. , , . Find ^{2} and
20. Show that every permutation of a finite set is a product of disjoint cycles.

21. Show that number of odd permutations in S_n is same as number of even permutations in S_n.
22. Find all the cosets of the subgroup 4Z of Z.
23. Find the index of <3> in the group Z_{24} .
24. Show that identity element is preserved under a group homomorphism.
25. Show that a group homomorphism is one to one iff Ker()=.
26. What are zero divisors of a ring. Which are the zero divisors of Z_6.

(10 x 4 = 40 Marks)

Section C

Answer ***any six*** questions. Each question carries 7 marks

27. Let S be the set of all real numbers except -1. Define Check whether is a binary operation on S. Which is the identity element in S?
28. How many groups are there of order 4 upto isomorphism. Which are they?
29. Prove that subgroup of a cyclic group is cyclic.
30. Draw the lattice diagram of Z_{18}.
31. Show that symmetries of an equilateral triangle form a group.
32. Show that no permutation in S_n can be expressed both as a product of an even number of transpositions and as a product an odd number of transpositions.
33. State and prove Lagrange’s theorem.
34. Define field. Give an example.
35. Show that upto isomorphism there exists only one infinite cyclic group.

(6 x 7 = 42 Marks)

Section D

Answer ***any two*** questions. Each question carries 13 marks

36. (a) Show that GL(n,R) forms a group under matrix multiplication.

    (b) Show that in a group G , $(ab)^{-1}= b^{-1}a^{-1}$ and $(a^{-1})^{-1}=a$ for all a,b G.
37. State and prove Cayley’s theorem.
38. Show that every finite integral domain is a field and every field is an integral domain.

(2 x 13 = 26 Marks)

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