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Target Setting for Inefficient DMUs for an Acceptable Level of Efficient Performance

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Abstract

Data Envelopment Analysis is a nonparametric technique used to rank similar decision making units by evaluating the relative efficiency. It can also be used to set target values for inefficient DMUs so as to perform efficiently. In common set of weights DEA model, a common set of weights for inputs and outputs for DMUs will be determined and this common set of weights is used to evaluate the relative efficiency and ranking of each DMU. The most commonly used method to generate common set of weights is the method based on the goal programming. In traditional models, target setting for inefficient DMUs is also considered so as to attain a level on the efficient frontier. Sometimes forcing a DMU to function so as to attain the maximum efficiency value may result in undesirable after effects. So it is very important to consider models which suggest target values for inefficient DMU so as attain an acceptable level of efficiency. This paper aims to consider such a target setting also.

Keywords: Data Envelopment Analysis (DEA), Multi-objective Optimization, Goal programming, Common set of weights model

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1. INTRODUCTION

Data Envelopment Analysis (DEA) is an LPP based nonparametric technique developed by Charnes et al [1] in 1978 to evaluate relative efficiency of similar decision making units (DMUs) that uses homogeneous multiple inputs to produce homogeneous multiple outputs. In traditional DEA models, each DMU can select the best input and output weights by solving an LP problem to achieve the highest efficiency. So if there are n DMUs, it is required to solve n LP problems to attain the weights for each DMUs [2, 3]. The drawbacks of these type of DEA models are explained in [4-7]. To overcome these shortcomings they developed common set of weights models. These models in the cited papers use the solution of one LP problem rather than the solutions of n LP problems to set the common set of weights. For this it is required to solve a multiple objective problem. The most commonly used method to solve a multiple objective LP problem is by goal programming. Using goal programming a multiple objective LP problem can be transformed to an LP problem. Some of the multiobjective models and goal programming models are given in [8-13].

Suppose there are n DMUs to be evaluated where each DMUj, $j = 1,2, \dots \dots . n$ consumes m inputs $x_{ij}, i = 1,2, \dots \dots . m$ to produce s outputs $y_{rj}, r = 1,2, \dots \dots . . s$. To solve goal programming problem to determine the common set of weights using Hosseinzadeh Lofti et al [10], consider the LP problem,

$$minimize \sum_{j=1}^{n} S_j$$

$subjec\ to$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + s_j = 0, \qquad j = 1,2, \dots \dots n$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1,2, \dots \dots n$$

$$s_j \ge 0, \qquad j = 1,2, \dots \dots n$$

$$u_r, v_i \ge \epsilon \qquad r = 1,2, \dots \dots s;\ i = 1,2, \dots \dots m \qquad \text{----------------} \quad \text{(M1)}$$

Since in the model $S_j \ge 0$, the second set of constraints is redundant and can be removed from the model.

Using the optimal solutions $\left(u_r^*, v_i^*, S_j^*\right) \forall i \,\&\, r,$ the efficiency score for DMUj is given by

$$\theta_j^* = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}} = 1 - \frac{S_j^*}{\sum_{i=1}^{m} v_i^* x_{ij}}, \qquad j = 1,2,\dots\dots.n \qquad \text{------------------} \quad (1)$$

Based on this efficiency value it is possible to rank the DMUs. All DMUs having $\theta_j^* = 1$ can be considered as efficient or non dominated DMUs.

This model is developed to rank the DMUs based on their relative efficiency values θ_j^*. This model is not applicable to suggest target values for the efficient functioning of inefficient DMUs or DMUs having $\theta_j^* < 1$. The proposed paper aims to modify the model to suggest target values for a satisfactory functioning of each DMU. It is proposed to consider the satisfactory level of efficiency by solving the LP problem of Hosseinzadeh Lofti et al [10] by introducing a fictitious DMU.

The rest of the paper is organized as follows. In section 2 the LP problem to determine the satisfactory efficiency and common set of weights is considered. Target setting for satisfactory functioning of inefficient DMUs is given section 3. Different target setting models are given in section 4 and example to illustrate the proposed models is given in section 5.

2. DETERMINATION OF SATISFACTORY LEVEL OF EFFICIENCY

In this section a satisfactory level of efficiency is proposed by considering a fictitious DMU which consumes the inputs that are the average of respective inputs of DMU_1 to DMUn and produces output as the average of the respective outputs of DMU_1 to DMU_n. Let us name the fictitious DMU as DMU_{n+1}. So by the assumption, the input-output factors $X_{i,n+1}$ and $Y_{r,n+1}$ of DMU_{n+1} are given by

$$X_{i,n+1} = \sum_{j=1}^{n} X_{ij}/n \qquad i = 1,2,\dots..m$$

$$Y_{r,n+1} = \sum_{j=1}^{n} Y_{rj} \ /n \qquad r = 1,2,\dots\dots.s$$

Include DMU_{n+1} into the set of DMUs and evaluate the efficiency as given below. Let θ_{n+1}^* denote the efficiency of DMU_{n+1}. The peculiarity of this efficiency value is that it considers all inputs of all DMUs and all outputs of all DMUs for the efficiency evaluation. So this efficiency has an importance in the performance evaluation of each DMU. If the efficiency value of a particular DMU is greater than θ^*_{n+1}, then it can be concluded that the performance of the DMU is superior to the average performance. So in the proposed model setting θ^*_{n+1} as the acceptable level of efficiency is justified. This level, θ_a is determined by solving the following LP problem.

$$minimize \sum_{j=1}^{n+1} S_j$$

Subject to

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + s_j = 0, \qquad j = 1,2, \dots \dots n+1$$

$$s_j \geq 0, \qquad j = 1,2, \dots \dots n+1$$

$$u_r, v_i \geq \epsilon \qquad r = 1,2, \dots \dots s; \qquad i = 1,2, \dots \dots m \qquad \text{-------------- (M2)}$$

Using the optimal solution $(u^*_r, v^*_i, S^*_{n+1})$, the acceptable efficiency θ_a is given by

$$\theta_a = 1 - \frac{S^*_{n+1}}{\sum_{i=1}^{m} v^*_i x_{i,n+1}}. \qquad \text{---------------------- (2)}$$

2.1 Observation

$0 < \theta_a \leq 1$ and $\theta_a = 1$ only when all the input and output values are same for all the DMUs

3. TARGET SETTING FOR INEFFICIENT DMUs

Let (u^*_r, v^*_i, S^*_j) be the optimal values of (M2). Let θ^*_j denote the efficiency value of DMUj, $j = 1,2, \dots \dots n$ evaluated by substituting (u^*_r, v^*_i, S^*_j) in (1). Using θ^*_j, $j = 1,2, \dots \dots n$ it is possible to classify the DMUs into two groups, DMUs having efficiency value less than θ_a, the acceptable efficiency and DMUs having efficiency value greater than or equal to θ_a.

3.1 Definition

The DMUs having efficiency value greater than or equal to θ_a are called acceptable efficient (a-efficient) DMUs and if the efficiency is unity then such a DMU is called completely efficient DMU.

If the efficiency value of a particular DMU is less than θ_a, then it is called an inefficient (a-inefficient) DMU.

3.2 Ranking and Target setting

This paper is aimed to rank the DMUs based on their performance as well as to suggest target values for inefficient DMUs so as to achieve an acceptable level of efficiency. Using the LP problem given in (M2), the optimal deviations S_j^*, $j = 1,2,\ldots\ldots n$ and the common set of weights u_r *and* v_i $r = 1,2,\ldots\ldots s$; $i = 1,2,\ldots\ldots m$ can be evaluated. By substituting the optimal deviations and the common set of weights in (1) the efficiency value for each DMU for $j = 1,2,\ldots\ldots n$ can obtained easily. These efficiency values can be used to rank the DMUs.

To suggest target values first of all it is required to identify the inefficient DMUs. Let DMUo be an inefficient DMU which can be identified using (M2). By solving an LP problem, the improvement in the values of each input and output for DMUo can be determined. Let α_{io} be the improvement required in the ith input of DMUo for $i = 1,2,\ldots\ldots m$ and β_{ro} be the improvement required for the rth output for $r = 1,2,\ldots\ldots s$. Then to determine α_{io} for $i = 1,2,\ldots\ldots m$ and β_{ro} for $r = 1,2,\ldots\ldots s$, the following LP problem is solved

$$minimize \sum_{i=1}^{m} \alpha_{io} + \sum_{r=1}^{s} \beta_{ro}$$

subject to

$$\theta_a\left(\sum_i v_i^*(x_{io} - \alpha_{io})\right) - \sum_r u_r^*(y_{ro} + \beta_{ro}) \leq 0$$

$$\sum_r u_r^*(y_{ro} + \beta_{ro}) - \sum_i v_i^*(x_{io} - \alpha_{io}) \leq 0$$

$$\alpha_{io} \leq x_{io},\ i = 1,2,\ldots\ldots m$$

$$\alpha_{io}, \beta_{ro} \geq 0 \ \ \forall i \ \& \ \forall r \qquad \text{-------------------------- (M4)}$$

Where θ_a is the acceptable efficiency determined using (2) and (M2), (u_r^*, v_i^*) are the common set of weights determined using (M2). The target input-output vector for DMUo, $\tilde{x}_{io}$ and $\tilde{y}_{ro}$ is then given by

$$\tilde{x}_{io} = x_{io} - \alpha_{io} \text{ and } \tilde{y}_{ro} = y_{ro} + \beta_{ro} \qquad \text{--------------------------- (3)}$$

The target $\tilde{x}_{io}$ is the maximum input value of i^{th} input to attain a-efficiency for DMUo. Similarly $\tilde{y}_{ro}$ is the minimum output value for the r^{th} output to attain a-efficiency for DMUo.

4. DIFFERENT MODELS FOR TARGET SETTING

This section of the paper is devoted to describe variants of the proposed model. The main advantage of the proposed model is that it is possible to modify the model so as to extend it to use for a wide variety of cases. Firstly, the model given in (M4) is extended to suggest target inputs/outputs values where certain of the input/output values are held fixed so that change is possible only in the remaining set of input/output values in order to achieve a-efficiency.

Suppose we do not want to consider a change in a particular input or output. Then we can design such a model in the following way. Suppose we do not want to revise the i_p^{th} input of the inefficient DMUo. Then we consider the LP problem

$$minimize \sum_{\substack{i=1 \\ i \neq p}}^{m} \alpha_{io} + \sum_{r=1}^{s} \beta_{ro}$$

subject to

$$\theta_a \left(\sum_{\substack{i \\ i \neq p}} v_i^* (x_{io} - \alpha_{io}) + v_p^* x_{po} \right) - \sum_r u_r^* (y_{ro} + \beta_{ro}) \leq 0$$

$$\sum_r u_r^* (\tilde{y}_{ro} + \beta_{ro}) - \left(\sum_{\substack{i \\ i \neq p}} v_i^* (x_{io} - \alpha_{io}) + v_p^* x_{po} \right) \leq 0$$

$$\alpha_{io} \leq x_{io}, \ \ i = 1,2, \dots \dots m, i \neq p$$

$\alpha_{io}, \beta_{ro} \geq 0 \;\; \forall i \;\&\; \forall r$ ---------------------------- (M5)

Similarly if we do not want to revise any particular output say r_q^{th} output. Then as above we can develop the LP model to suggest target values. By extending these models it is possible to develop the input oriented as well as the output oriented models. To develop input oriented or output oriented model for target setting, we can directly modify (M4). In case of output oriented target setting we will keep the given output and will try to suggest corresponding input. However in the case of input oriented model, we will keep the given input fixed and will suggest modified output for efficiency.

However the target setting using an LP problem has some limitations. As in the example, we can see that the target suggestion using (M4) and (M5) may concentrate on a single component or some particular components of input or output for the improvement. But in real world situation this may not be a correct mechanism. We can simply modify our model to accommodate the situation. Suppose we need the utilization of the input-output factors in a particular ratio for the improvement, then we can include it in our model as constraints. The main advantage of this discussion is that the proposed model can concentrate a particular input/output for the improvement or it can revise all inputs and outputs with equal importance based on a predefined ratio. This model is hence giving more power for the decision maker. The following section explains the models with an example.

5. EXAMPLE

Table 1 (Given data on 5 DMUs with 2 inputs and 2 outputs)

	X1	X2	Y1	Y2
DMU1	6	5	9	12
DMU2	4	2	10	20
DMU3	3	7	12	15
DMU4	6	8	14	16
DMU5	2	10	18	4

Consider the example (given in Subhash C. Ray) of 5 DMUs having 2 inputs and 2 outputs. Data is given in Table 1. The Dummy DMU of our discussion is given in Table2.

Table 2(Data for fictitious DMU)

	X1	X2	Y1	Y2
Dummy DMU	4.2	6.4	12.6	13.4

The LP problem given in (M2) corresponding to the data is given by

$$minimize \sum_{j=1}^{6} S_j$$

$$9u_1 + 12u_2 - 6v_1 - 5v_2 + S_1 = 0$$

$$10u_1 + 20u_2 - 4v_1 - 2v_2 + S_2 = 0$$

$$12u_1 + 15u_2 - 3v_1 - 7v_2 + S_3 = 0$$

$$14u_1 + 16u_2 - 6v_1 - 8v_2 + S_4 = 0$$

$$18u_1 + 4u_2 - 2v_1 - 10v_2 + S_5 = 0$$

$$12.6u_1 + 13.4u_2 - 4.2v_1 - 6.4v_2 + S_6 = 0$$

$$u_r, v_i \geq \varepsilon \quad \text{for } r = 1,2 \text{ and } i = 1,2$$

$$S_j \geq 0 \text{ for } j = 1,2, \dots .5$$

Corresponding to $\varepsilon = 0.01$

We get $(u_1, u_2, v_1, v_2) = (0.0115,\ 0.01,\ 0.0738,\ 0.01)$

From table 3 we get the acceptable level of efficiency $\theta_a = 0.75$, DMU_2 and DMU_5 are completely efficient, DMU_1 and DMU_4 are inefficient.

Table 3(Efficiency and optimal slacks)

DMUs	1	2	3	4	5	Dummy DMU
S_j^*	0.2692	0	0.0031	0.2015	0	0.0948
θ_j^*	0.45	1	0.99	0.61	1	0.75

The target values for the inefficient DMUs are given in Table 4

Table 4(Target input-output values for inefficient DMUs)

	Basic model				Output oriented				Input oriented			
	x_{1j}	x_{2j}	y_{1j}	y_{2j}	x_{1j}	x_{2j}	y_{1j}	y_{2j}	x_{1j}	x_{2j}	y_{1j}	y_{2j}
DMU_1	3.38	5	9	12	6	5	21.56	12	3.38	5	9	12
DMU_4	4.74	8	14	16	6	8	20.03	16	4.74	8	14	16

From Tables 4, it is clear that the target setting is suggested by concentrating on a particular factor. This we can overcome by imposing additional constrains based on the importance and availability of input-output factors. For example if we impose additional constraints $3\alpha_{1j} \leq \alpha_{2j}, \beta_{1j} \leq 2\beta_{2j}$, $\alpha_{1j} + \alpha_{2j} \leq \beta_{1j} + \beta_{2j}$ for $j = 1, 4$. Then the target values of inefficient DMUs with $\theta_a = 0.75$ for the basic model is given in Table 5.

Table 5(Target input-output values for inefficient DMUs)

	x_{1j}	x_{2j}	y_{1j}	y_{2j}
DMU_1	4.81	1.44	12.17	13.58
DMU_4	5.43	6.29	15.52	16.76

6. CONCLUSION

The method discussed in this paper can be used to classify the given DMUs into two categories efficient and inefficient. It also can be used to define an acceptable level of efficiency and suggest target values to attain the acceptable level of efficiency. The main advantage of the model is that it is adaptable to a wide variety of situations like input/output weight restriction, restriction on changes in certain inputs/outputs etc. so this model helps the decision maker to make analysis on the performance of DMUs in many different ways.

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