Boxdot and Star Products on Interval - Valued Intuitionistic Fuzzy Graphs

Tintumol Sunny, Sr. Magie Jose

Abstract: In this paper, Boxdot product and Star product on interval–valued intuitionistic fuzzy graphs has been introduced and degree of vertices of these new product graphs are determined. Some results involving these products are stated and proved.

Keywords: Interval-valued intuitionistic fuzzy graph, Strong interval - valued intuitionistic fuzzy graph, Boxdot product, Star product, degree of vertices.

I. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh[7] in 1965 for defining uncertainty. In 1975, Zadeh[8] introduced the notion of interval-valued fuzzy sets as an extension of fuzzy sets, in which the values of the membership degrees are intervals of numbers instead of the numbers. In 1986, Atanassov introduced Intuitionistic Fuzzy Sets [4] which provides the opportunity to model the problem precisely based on the existing information and observations. After three years Atanassov and Gargov[5] proposed Interval-Valued Intuitionistic Fuzzy set (IVIFS) which is helpful to model the problem more accurately. The fuzzy graph theory was first introduced by Rosenfeld [10] in 1975. Yeh and Bang [12] also introduced fuzzy graphs independently. Fuzzy graphs are useful to represent relationships which deal with uncertainty. It has numerous applications to problems in various fields. Interval-Valued Fuzzy Graphs (IVFG) are defined by Akram and Dudek[2] in 2011. Atanassov[6] introduced the concept of intuitionistic fuzzy relations and Intuitionistic Fuzzy Graph (IFG). ShovanDogra [11] introduced different types of products of fuzzy graphs. S.N.Mishra and A.Pal[9] introduced the product of interval valued intuitionistic fuzzy graph. Akram and BijanDavvaz[1] introduced Strong Intuitionistic Fuzzy Graphs (SIFG). The notions of Strong Interval-Valued Intuitionistic Fuzzy Graphs (SIVIFG) are introduced by A.MohamedIsmayil andA.Mohamed Ali [3]. This paper has been organized as follows. Preliminaries required for this study are given in section 2. In section 3 and 4, Boxdot and Star product on interval-valued intuitionistic fuzzy graphs has been defined and some of its properties are discussed.

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II. PRELIMINARIES

Definition 2.1[10]A fuzzy graph$G$is a pair of functions $G = (\sigma, \mu)$ where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation onσ. The underlying crisp graph of$G = (\sigma, \mu)$ is denoted by$G^* = (V, E)$where$E \subseteq V \times V$.
Let $D[0, 1]$ be the set of all closed subintervals of the interval $[0, 1]$. If $M \in D[0,1]$then it can be represented as$M = [M_L, M_U]$, where M_L and $M_U$are the lower and upper limits of M.

Definition 2.2[6]Anintuitionistic fuzzy graph with underlying set V is defined to be a pair $G = (\sigma, \mu)$ where

1. the functions $M_\sigma: V \to D[0, 1]$ and $N_\sigma: V \to D[0, 1]$denote the degree of membership and non membership of the element $x \in V$, respectively, such that $0 \le M_\sigma(x) + N_\sigma(x) \le 1$for all $x \in V$.
2. the functions $M_\mu: E \subseteq V \times V \to D[0, 1]$ and $N_\mu: E \subseteq V \times V \to D[0, 1]$are defined by
$M_\mu((x, y)) \le min(M_\sigma(x),\ M_\sigma(y))$and
$N_\mu((x, y)) \ge max\left(N_\sigma(x),\ N_\sigma(y)\right)$such that
$$0 \le M_\mu((x, y)) + N_\mu((x, y)) \le 1, \forall (x, y) \epsilon E.$$

Definition 2.3 [1]An intuitionistic fuzzy graph G = (σ, μ) is called strong intuitionistic fuzzy graph if $M_\mu((x, y)) = min(M_\sigma(x), M_\sigma(y))$and $N_\mu((x, y)) = max(N_\sigma(x), N_\sigma(y))$, $\forall (x, y) \epsilon E$.

Definition 2.4[2]An interval - valued intuitionistic fuzzy graph with underlying set V is defined to be a pair $G = (\sigma, \mu)$ where,

1. the functions $M_\sigma: V \to D[0, 1]$and $N_\sigma: V \to D[0, 1]$denote the degree of membership and non membership of the element $x \in V$,respectively, such that $0 \le M_\sigma(x) + N_\sigma(x) \le 1$, $\forall x \in V$.
2. the functions $M_\mu: E \subseteq V \times V \to D[0, 1]$ and $N_\mu: E \subseteq V \times V \to D[0, 1]$ are defined by
$M_{\mu L}((x, y)) \le min(M_{\sigma L}(x), M_{\sigma L}(y))$
$N_{\mu L}((x, y)) \ge max(N_{\sigma L}(x), N_{\sigma L}(y))$
$M_{\mu U}\left((x, y)\right) \le min\left(M_{\sigma U}(x),\ M_{\sigma U}(y)\right)andN_{\mu U}\left((x, y)\right) \ge max\left(N_{\sigma U}(x),\ N_{\sigma U}(y)\right)$ such that
$$0 \le M_{\mu U}\left((x, y)\right) + N_{\mu U}\left((x, y)\right) \le 1, \quad \forall (x, y) \epsilon E.$$

Definition 2.5[3]An interval valued intuitionistic fuzzy graph$G = (\sigma, \mu)$ is called strong interval valued intuitionistic fuzzy graph if

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$$M_{\mu L}((x,y)) = min(M_{\sigma L}(x), \quad M_{\sigma L}(y))$$
$$N_{\mu L}((x,y)) = max(N_{\sigma L}(x), N_{\sigma L}(y))$$
$$M_{\mu U}((x,y)) = min(M_{\sigma U}(x), M_{\sigma U}(y))$$
$$N_{\mu U}((x,y)) = max(N_{\sigma U}(x), N_{\sigma U}(y)), \quad \forall (x,y)\epsilon E$$

Definition 2.6[9]Let $G = (\sigma,\mu)$ be an interval valued intuitionistic fuzzy graph. For any vertex $x \in V$, degree of the vertex x is defined as an ordered pair $(d_G^-(x), d_G^+(x))$ where

$$d_G^-(x) = \sum_{y\in V: xy\in E} M_{\mu L}((x,y)) - \sum_{y\in V: xy\in E} N_{\mu L}((x,y))$$

and

$$d_G^+(x) = \sum_{y\in V: xy\in E} M_{\mu U}((x,y)) - \sum_{y\in V: xy\in E} N_{\mu U}((x,y))$$

Definition 2.7 Let $\sigma_1 = ([M_{\sigma_1 L}, M_{\sigma_1 U}], [N_{\sigma_1 L}, N_{\sigma_1 U}])$and $\sigma_2 = ([M_{\sigma_2 L}, M_{\sigma_2 U}], [N_{\sigma_2 L}, N_{\sigma_2 U}])$be interval valued intuitionistic fuzzy subsets of $V_1(G_1)$ and $V_2(G_2)$ respectively. Then$\sigma_1 \le \sigma_2$if and only if

$$M_{\sigma_1 L}(x) \le M_{\sigma_2 L}(y),\ M_{\sigma_1 U}(x) \le M_{\sigma_2 U}(y)$$

and$N_{\sigma_1 L}(x) \ge N_{\sigma_2 L}(y),\ N_{\sigma_1 U}(x) \ge N_{\sigma_2 U}(y)$,
$\forall x \in V_1$and$\forall y \in V_2$

III. BOXDOT PRODUCT ON INTERVAL-VALUED INTUITIONISTIC FUZZY GRAPHS

Definition 3.1 Let$\sigma_1$and $\sigma_2$be interval-valued intuitionistic fuzzy subsets of $V_1$and V_2 respectively. Let μ_1 and $\mu_2$be interval-valued intuitionistic fuzzy subsets of E_1 and E_2 respectively. Then Boxdot product$G_1 \boxdot G_2$ of the two strong interval -valued intuitionistic fuzzy graphs $G_1 = (\sigma_1, \mu_1)$ and G_2=$(\sigma_2, \mu_2)$is defined as a pair $(\sigma_1 \boxdot \sigma_2, \mu_1 \boxdot \mu_2)$where $\sigma 1\boxdot\sigma 2 = M\sigma 1L\boxdot M\sigma 2L$, $M\sigma 1U\boxdot M\sigma 2U$, $N\sigma 1L\boxdot N\sigma 2L$, $N\sigma 1U\boxdot N\sigma 2U$and $\mu 1\boxdot\mu 2 = M\mu 1L\boxdot M\mu 2L$, $M\mu 1U\boxdot M\mu 2U$, $N\mu 1L\boxdot N\mu 2L$, $N\mu 1U\boxdot N\mu 2U$ are interval-valued intuitionistic fuzzy sets on $V = V_1 \boxdot V_2$ and $E = E_1 \boxdot E_2 = \{(x_1,x_2)(y_1,y_2): x_1 = y_1, x_2y_2 \notin E2\ or\ x1y1\in E1\&x2y2\notin E2$ respectively, which satisfy the following
properties

$$\left.\begin{array}{l}(M_{\sigma_1 L} \boxdot M_{\sigma_2 L})(x,y) = min\{M_{\sigma_1 L}(x), M_{\sigma_2 L}(y)\}\\ (M_{\sigma_1 U} \boxdot M_{\sigma_2 U})(x,y) = min\{M_{\sigma_1 U}(x), M_{\sigma_2 U}(y)\}\\ (N_{\sigma_1 L} \boxdot N_{\sigma_2 L})(x,y) = max\{N_{\sigma_1 L}(x), N_{\sigma_2 L}(y)\}\\ (N_{\sigma_1 U} \boxdot N_{\sigma_2 U})(x,y) = max\{N_{\sigma_1 U}(x), N_{\sigma_2 U}(y)\}\end{array}\right\}$$

$$\forall\ (x,y) \in V_1 \boxdot V_2$$

$$\left.\begin{array}{l}(M_{\mu_1 L} \boxdot M_{\mu_2 L})((x_1,x_2)(y_1,y_2))\\ = min\{M_{\sigma_1 L}(x_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}\\ (M_{\mu_1 U} \boxdot M_{\mu_2 U})((x_1,x_2)(y_1,y_2))\\ = min\{M_{\sigma_1 U}(x_1), M_{\sigma_2 U}(x_2), M_{\sigma_2 U}(y_2)\}\\ (N_{\mu_1 L} \boxdot N_{\mu_2 L})((x_1,x_2)(y_1,y_2))\\ = max\{N_{\sigma_1 L}(x_1), N_{\sigma_2 L}(x_2), N_{\sigma_2 L}(y_2)\}\\ (N_{\mu_1 U} \boxdot N_{\mu_2 U})((x_1,x_2)(y_1,y_2))\\ = max\{N_{\sigma_1 U}(x_1), N_{\sigma_2 U}(x_2), N_{\sigma_2 U}(y_2)\}\end{array}\right\}$$

if$x_1 = y_1 \in V_1, x_2y_2 \notin E_2$

$$\left.\begin{array}{l}(M_{\mu_1 L} \boxdot M_{\mu_2 L})((x_1,x_2)(y_1,y_2)) =\\ \quad min\{M_{\mu_1 L}(x_1,y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}\\ (M_{\mu_1 U} \boxdot M_{\mu_2 U})((x_1,x_2)(y_1,y_2)) =\\ \quad min\{M_{\mu_1 U}(x_1,y_1), M_{\sigma_2 U}(x_2), M_{\sigma_2 U}(y_2)\}\\ (N_{\mu_1 L} \boxdot N_{\mu_2 L})((x_1,x_2)(y_1,y_2)) =\\ \quad max\{N_{\mu_1 L}(x_1,y_1), N_{\sigma_2 L}(x_2), N_{\sigma_2 L}(y_2)\}\\ (N_{\mu_1 U} \boxdot N_{\mu_2 U})((x_1,x_2)(y_1,y_2)) =\\ \quad max\{N_{\mu_1 U}(x_1,y_1), N_{\sigma_2 U}(x_2), N_{\sigma_2 U}(y_2)\}\end{array}\right\}$$

if$x_1y_1 \in E_1 \& x_2y_2 \notin E_2$

Proposition 3.1 If G_1 and G_2 are strong interval-valued intuitionistic fuzzy graphs, then the Boxdot Product of G_1 and G_2, $G_1 \boxdot G_2$ is also a strong interval -valued intuitionistic fuzzy graphs.

*Proof :*Let G_1 and G_2 are strong interval-valued intuitionistic fuzzy graphs.

Hence $\forall\ x_i, y_i \in E_i, i = 1, 2$

$$M_{\mu_i L}((x_i,y_i)) = min\left(M_{\sigma_i L}(x_i), M_{\sigma_i L}(y_i)\right) M_{\mu_i U}((x_i,y_i)) = min\left(M_{\sigma_i U}(x_i), \quad M_{\sigma_i U}(y_i)\right)$$
$$N_{\mu_i L}((x_i,y_i)) = max(N_{\sigma_i L}(x_i), N_{\sigma_i L}(y_i))$$
$$N_{\mu_i U}((x_i,y_i)) = max(N_{\sigma_i U}(x_i), N_{\sigma_i U}(y_i))$$

Here

$$E = E_1 \boxdot E_2 = \left\{\begin{array}{c}(x_1,x_2)(y_1,y_2): x_1 = y_1, x_2y_2 \in E_2\\ or\ x_1y_1 \in E_1, x_2y_2 \notin E_2\end{array}\right\}$$

Let $(x_1,x_2)(y_1,y_2) \in E$

Case I: $x_1 = y_1, x_2y_2 \notin E_2$
Consider $(M_{\mu_1 L} \boxdot M_{\mu_2 L})((x_1,x_2)(y_1,y_2))$

$$= min\{M_{\sigma_1 L}(x_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$
$$= min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

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$$min\left\{\left(M_{\sigma_1 L}\boxdot M_{\sigma_2 L}\right)(x_1,x_2),\left(M_{\sigma_1 L}\boxdot M_{\sigma_2 L}\right)(y_1,y_2)\right\}$$
$$= min\left\{\begin{matrix} min\{M_{\sigma_1 L}(x_1), \quad M_{\sigma_2 L}(x_2)\}, \\ min\{M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(y_2)\}\end{matrix}\right\}$$
$$= min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

i.e.,

$$\left(M_{\mu_1 L}\boxdot M_{\mu_2 L}\right)((x_1,x_2)(y_1,y_2))$$
$$= min\left\{\left(M_{\sigma_1 L}\boxdot M_{\sigma_2 L}\right)(x_1,x_2),\left(M_{\sigma_1 L}\boxdot M_{\sigma_2 L}\right)(y_1,y_2)\right\}$$

Similarly, it can be proved that

$$\left(M_{\mu_1 U}\boxdot M_{\mu_2 U}\right)((x_1,x_2)(y_1,y_2))$$
$$= min\left\{\left(M_{\sigma_1 U}\boxdot M_{\sigma_2 U}\right)(x_1,x_2),\left(M_{\sigma_1 U}\boxdot M_{\sigma_2 U}\right)(y_1,y_2)\right\}$$
$$\left(N_{\mu_1 L}\boxdot N_{\mu_2 L}\right)((x_1,x_2)(y_1,y_2))$$
$$= max\left\{\left(N_{\sigma_1 L}\boxdot N_{\sigma_2 L}\right)(x_1,x_2),\left(N_{\sigma_1 L}\boxdot N_{\sigma_2 L}\right)(y_1,y_2)\right\}$$
$$\left(N_{\mu_1 U}\boxdot N_{\mu_2 U}\right)((x_1,x_2)(y_1,y_2))$$
$$= max\left\{\left(N_{\sigma_1 U}\boxdot N_{\sigma_2 U}\right)(x_1,x_2),\left(N_{\sigma_1 U}\boxdot N_{\sigma_2 U}\right)(y_1,y_2)\right\}$$

Hence in this case $G_1 \boxdot G_2$ is a strong interval-valued intuitionistic fuzzy graph.

Case II: $x_1y_1 \in E_1 \& x_2y_2 \notin E_2$

Consider $\left(M_{\mu_1 L}\boxdot M_{\mu_2 L}\right)((x_1,x_2)(y_1,y_2))$

$$= min\left\{M_{\mu_1 L}(x_1,y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\right\}$$
$$= min\left\{\begin{matrix} min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1)\}, \\ M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\end{matrix}\right\}$$
$$= min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

and

$$min\left\{\left(M_{\sigma_1 L}\boxdot M_{\sigma_2 L}\right)(x_1,x_2),\left(M_{\sigma_1 L}\boxdot M_{\sigma_2 L}\right)(y_1,y_2)\right\}$$
$$= min\left\{\begin{matrix} min\{M_{\sigma_1 L}(x_1), \quad M_{\sigma_2 L}(x_2)\}, \\ min\{M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(y_2)\}\end{matrix}\right\}$$
$$= min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

i.e.,

$$\left(M_{\mu_1 L}\boxdot M_{\mu_2 L}\right)((x_1,x_2)(y_1,y_2))$$
$$= min\left\{\left(M_{\sigma_1 L}\boxdot M_{\sigma_2 L}\right)(x_1,x_2),\left(M_{\sigma_1 L}\boxdot M_{\sigma_2 L}\right)(y_1,y_2)\right\}$$

Similarly, it can be proved that

$$\left(M_{\mu_1 U}\boxdot M_{\mu_2 U}\right)((x_1,x_2)(y_1,y_2))$$
$$= min\left\{\left(M_{\sigma_1 U}\boxdot M_{\sigma_2 U}\right)(x_1,x_2),\left(M_{\sigma_1 U}\boxdot M_{\sigma_2 U}\right)(y_1,y_2)\right\}$$
$$\left(N_{\mu_1 L}\boxdot N_{\mu_2 L}\right)((x_1,x_2)(y_1,y_2))$$
$$= max\left\{\left(N_{\sigma_1 L}\boxdot N_{\sigma_2 L}\right)(x_1,x_2),\left(N_{\sigma_1 L}\boxdot N_{\sigma_2 L}\right)(y_1,y_2)\right\}$$
$$\left(N_{\mu_1 U}\boxdot N_{\mu_2 U}\right)((x_1,x_2)(y_1,y_2))$$
$$= max\left\{\left(N_{\sigma_1 U}\boxdot N_{\sigma_2 U}\right)(x_1,x_2),\left(N_{\sigma_1 U}\boxdot N_{\sigma_2 U}\right)(y_1,y_2)\right\}$$

Hencein this case also $G_1 \boxdot G_2$ is a strong interval-valued intuitionistic fuzzy graph.

Proposition 3.2 If $G_1 \boxdot G_2$ is a strong interval-valued intuitionistic fuzzy graph then atleast G_1 or G_2 must be strong.

Proof : Suppose that G_1 and G_2 are not strong interval-valued intuitionistic fuzzy graphs. So there exists $x_i, y_i \in E_i$, $i = 1, 2$ such that

$$M_{\mu_i L}((x_i,y_i)) < min\left(M_{\sigma_i L}(x_i), M_{\sigma_i L}(y_i)\right) M_{\mu_i U}((x_i,y_i)) < min\left(M_{\sigma_i U}(x_i), \quad M_{\sigma_i U}(y_i)\right)$$
$$N_{\mu_i L}((x_i,y_i)) > max\left(N_{\sigma_i L}(x_i), N_{\sigma_i L}(y_i)\right)$$
$$N_{\mu_i U}((x_i,y_i)) > max\left(N_{\sigma_i U}(x_i), N_{\sigma_i U}(y_i)\right)$$

Here

$$E = E_1 \boxdot E_2 = \left\{\begin{matrix}(x_1,x_2)(y_1,y_2): x_1 = y_1, x_2y_2 \notin E_2 \\ or\ x_1y_1 \in E_1 x_2y_2 \notin E_2\end{matrix}\right\}$$

Let $(x_1,x_2)(y_1,y_2) \in E$.
If $x_1y_1 \in E_1 \& x_2y_2 \notin E_2$, then

$$\left(M_{\mu_1 L}\boxdot M_{\mu_2 L}\right)((x_1,x_2)(y_1,y_2))$$
$$= min\left\{M_{\mu_1 L}(x_1,y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\right\}$$
$$< min\left\{\begin{matrix} min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1)\}, \\ M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\end{matrix}\right\}$$
$$< min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

and

$$min\left\{\left(M_{\sigma_1 L}\boxdot M_{\sigma_2 L}\right)(x_1,x_2),\left(M_{\sigma_1 L}\boxdot M_{\sigma_2 L}\right)(y_1,y_2)\right\}$$
$$= min\left\{\begin{matrix} min\{M_{\sigma_1 L}(x_1), \quad M_{\sigma_2 L}(x_2)\}, \\ min\{M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(y_2)\}\end{matrix}\right\}$$
$$= min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

i.e.,

$$\left(M_{\mu_1 L}\boxdot M_{\mu_2 L}\right)((x_1,x_2)(y_1,y_2))$$
$$< min\left\{\left(M_{\sigma_1 L}\boxdot M_{\sigma_2 L}\right)(x_1,x_2),\left(M_{\sigma_1 L}\boxdot M_{\sigma_2 L}\right)(y_1,y_2)\right\}$$

Similarly it can be proved that

$$\left(M_{\mu_1 U}\boxdot M_{\mu_2 U}\right)((x_1,x_2)(y_1,y_2))$$
$$< min\left\{\left(M_{\sigma_1 U}\boxdot M_{\sigma_2 U}\right)(x_1,x_2),\left(M_{\sigma_1 U}\boxdot M_{\sigma_2 U}\right)(y_1,y_2)\right\}$$
$$\left(N_{\mu_1 L}\boxdot N_{\mu_2 L}\right)((x_1,x_2)(y_1,y_2))$$
$$> max\left\{\left(N_{\sigma_1 L}\boxdot N_{\sigma_2 L}\right)(x_1,x_2),\left(N_{\sigma_1 L}\boxdot N_{\sigma_2 L}\right)(y_1,y_2)\right\}$$
$$\left(N_{\mu_1 U}\boxdot N_{\mu_2 U}\right)((x_1,x_2)(y_1,y_2))$$
$$> max\left\{\left(N_{\sigma_1 U}\boxdot N_{\sigma_2 U}\right)(x_1,x_2),\left(N_{\sigma_1 U}\boxdot N_{\sigma_2 U}\right)(y_1,y_2)\right\}$$

i.e., $G_1 \boxdot G_2$ is not a strong interval-valued intuitionistic fuzzy graph, which is a contradiction.
Hence the proof.

Definition 3.2 For any vertex $(x_1,x_2) \in V_1 \boxdot V_2$ in $G_1 \boxdot G_2$, degree of the vertex (x_1,x_2) is defined as an ordered pair $\left(d^-_{G_1\boxdot G_2}(x_1,x_2), d^+_{G_1\boxdot G_2}(x_1,x_2)\right)$ where

$d^{-}_{G_1 \boxdot G_2}(x_1, x_2) =$

$$\left(\sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1 = y_1 \in V_1,\, x_2y_2 \notin E_2}} min\{M_{\sigma_1 L}(x_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\} + \sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1y_1 \in E_1, x_2y_2 \notin E_2}} min\{M_{\mu_1 L}(x_1, y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\} \right)$$

$$- \left(\sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1 = y_1 \in V_1,\, x_2y_2 \notin E_2}} max\{N_{\sigma_1 L}(x_1), N_{\sigma_2 L}(x_2), N_{\sigma_2 L}(y_2)\} + \sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1y_1 \in E_1, x_2y_2 \notin E_2}} max\{N_{\mu_1 L}(x_1, y_1), N_{\sigma_2 L}(x_2), N_{\sigma_2 L}(y_2)\} \right)$$

and

$d^{+}_{G_1 \boxdot G_2}(x_1, x_2) =$

$$\left(\sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1 = y_1 \in V_1,\, x_2y_2 \notin E_2}} min\{M_{\sigma_1 U}(x_1), M_{\sigma_2 U}(x_2), M_{\sigma_2 U}(y_2)\} + \sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1y_1 \in E_1, x_2y_2 \notin E_2}} min\{M_{\mu_1 U}(x_1, y_1), M_{\sigma_2 U}(x_2), M_{\sigma_2 U}(y_2)\} \right)$$

$$- \left(\sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1 = y_1 \in V_1,\, x_2y_2 \notin E_2}} max\{N_{\sigma_1 U}(x_1), N_{\sigma_2 U}(x_2), N_{\sigma_2 U}(y_2)\} + \sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1y_1 \in E_1, x_2y_2 \notin E_2}} max\{N_{\mu_1 U}(x_1, y_1), N_{\sigma_2 U}(x_2), N_{\sigma_2 U}(y_2)\} \right)$$

Proposition 3.3 Let σ_1 and σ_2 be interval valued intuitionistic fuzzy subsets of V_1 and V_2 respectively where $\sigma_1 \leq \sigma_2$ and μ_1 and μ_2 be interval valued intuitionistic fuzzy subsets of E_1 and E_2 respectively. Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ and $G = G_1 \boxdot G_2$. Then the following equalities hold.

$d^{-}_{G_1 \boxdot G_2}(x_1, x_2) =$

$$\left(\sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1 = y_1 \in V_1,\, x_2y_2 \notin E_2}} M_{\sigma_1 L}(x_1) + \sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1y_1 \in E_1, x_2y_2 \notin E_2}} M_{\mu_1 L}(x_1, y_1) \right) - \left(\sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1 = y_1 \in V_1,\, x_2y_2 \notin E_2}} N_{\sigma_1 L}(x_1) + \sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1y_1 \in E_1, x_2y_2 \notin E_2}} N_{\mu_1 L}(x_1, y_1) \right)$$

and

$d^{+}_{G_1 \boxdot G_2}(x_1, x_2) =$

$$\left(\sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1 = y_1 \in V_1,\, x_2y_2 \notin E_2}} M_{\sigma_1 U}(x_1) + \sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1y_1 \in E_1, x_2y_2 \notin E_2}} M_{\mu_1 U}(x_1, y_1) \right) - \left(\sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1 = y_1 \in V_1,\, x_2y_2 \notin E_2}} N_{\sigma_1 U}(x_1) + \sum_{\substack{(y_1,y_2)\in V_1 \boxdot V_2: \\ x_1y_1 \in E_1, x_2y_2 \notin E_2}} N_{\mu_1 U}(x_1, y_1) \right)$$

Proposition 3.4 Let σ_1 and σ_2 be interval valued intuitionistic fuzzy subsets of V_1 and V_2 respectively where $\sigma_1 \leq \sigma_2$ and μ_1 and μ_2 be interval valued intuitionistic fuzzy subsets of E_1 and E_2 respectively. Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be complete interval valued intuitionistic fuzzy graphs and $G = G_1 \boxdot G_2$. Then $d^{-}_{G_1 \boxdot G_2}(x_1, x_2) = 0$ and $d^{+}_{G_1 \boxdot G_2}(x_1, x_2) = 0$.

IV. STAR PRODUCT ON INTERVAL-VALUED INTUITIONISTIC FUZZY GRAPHS

Definition 4.1 Let σ_1 and σ_2 be interval-valued intuitionistic fuzzy subsets of V_1 and V_2 respectively. Let μ_1 and μ_2 be interval-valued intuitionistic fuzzy subsets of E_1 and E_2 respectively. Then Star product $G_1 \star G_2$ of the two strong interval -valued intuitionistic fuzzy graphs $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ is defined as a pair $(\sigma_1 \star \sigma_2, \mu_1 \star \mu_2)$ where $\sigma_1 \star \sigma_2 = ([M_{\sigma_1 L} \star M_{\sigma_2 L}, M_{\sigma_1 U} \star M_{\sigma_2 U}], [N_{\sigma_1 L} \star N_{\sigma_2 L},$ $N\sigma1U\star N\sigma2U$ and $\mu1\star\mu2=M\mu1L\star M\mu2L,$ $M\mu1U\star M\mu2U,$ $N\mu1L\star N\mu2L,$ $N\mu1U\star N\mu2U$ are interval-valued intuitionistic fuzzy sets on $V = V_1 \star V_2$ and $E = E_1 \star E_2 = \{(x_1, x_2)(y_1, y_2): x_1 = y_1, x_2y_2 \notin E_2 \text{ or } x_1y_1 \in$ $E1\&x2y2\in E2$ respectively, which satisfy the following properties

$$\left.\begin{matrix} (M_{\sigma_1 L} \star M_{\sigma_2 L})(x, y) = min\{M_{\sigma_1 L}(x), M_{\sigma_2 L}(y)\} \\ (M_{\sigma_1 U} \star M_{\sigma_2 U})(x, y) = min\{M_{\sigma_1 U}(x), M_{\sigma_2 U}(y)\} \\ (N_{\sigma_1 L} \star N_{\sigma_2 L})(x, y) = max\{N_{\sigma_1 L}(x), N_{\sigma_2 L}(y)\} \\ (N_{\sigma_1 U} \star N_{\sigma_2 U})(x, y) = max\{N_{\sigma_1 U}(x), N_{\sigma_2 U}(y)\} \end{matrix}\right\} \forall\, (x, y) \in V_1 \star V_2$$

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$$\left.\begin{aligned}
&\left(M_{\mu_1 L} \star M_{\mu_2 L}\right)((x_1,x_2)(y_1,y_2)) \\
&= min\{M_{\sigma_1 L}(x_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\} \\
&\left(M_{\mu_1 U} \star M_{\mu_2 U}\right)((x_1,x_2)(y_1,y_2)) \\
&= min\{M_{\sigma_1 U}(x_1), M_{\sigma_2 U}(x_2), M_{\sigma_2 U}(y_2)\} \\
&\left(N_{\mu_1 L} \star N_{\mu_2 L}\right)((x_1,x_2)(y_1,y_2)) \\
&= max\{N_{\sigma_1 L}(x_1), N_{\sigma_2 L}(x_2), N_{\sigma_2 L}(y_2)\} \\
&\left(N_{\mu_1 U} \star N_{\mu_2 U}\right)((x_1,x_2)(y_1,y_2)) \\
&= max\{N_{\sigma_1 U}(x_1), N_{\sigma_2 U}(x_2), N_{\sigma_2 U}(y_2)\}
\end{aligned}\right\} \text{if} x_1 = y_1 \in V_1, x_2 y_2 \notin E_2$$

$$\left.\begin{aligned}
&\left(M_{\mu_1 L} \star M_{\mu_2 L}\right)((x_1,x_2)(y_1,y_2)) \\
&= min\left\{M_{\mu_1 L}(x_1,y_1), M_{\mu_2 L}(x_2,y_2)\right\} \\
&\left(M_{\mu_1 U} \star M_{\mu_2 U}\right)((x_1,x_2)(y_1,y_2)) \\
&= min\left\{M_{\mu_1 U}(x_1,y_1), M_{\mu_2 U}(x_2,y_2)\right\} \\
&\left(N_{\mu_1 L} \star N_{\mu_2 L}\right)((x_1,x_2)(y_1,y_2)) \\
&= max\left\{N_{\mu_1 L}(x_1,y_1), N_{\mu_2 L}(x_2,y_2)\right\} \\
&\left(N_{\mu_1 U} \star N_{\mu_2 U}\right)((x_1,x_2)(y_1,y_2)) \\
&= max\left\{N_{\mu_1 U}(x_1,y_1), N_{\mu_2 U}(x_2,y_2)\right\}
\end{aligned}\right\}$$

$$\text{if} x_1 y_1 \in E_1 \& x_2 y_2 \in E_2$$

Proposition 4.1 If G_1 and G_2 are strong interval-valued intuitionistic fuzzy graphs, then the Stardot Product of G_1 and G_2, $G_1 \star G_2$ is also a strong interval -valued intuitionistic fuzzy graphs.

Proposition 4.2 If $G_1 \star G_2$ is a strong interval-valued intuitionistic fuzzy graph then atleast G_1 or G_2 must be strong.

Definition 4.2 For any vertex $(x_1,x_2) \in V_1 \star V_2$ in $G_1 \star G_2$, degree of the vertex (x_1,x_2) is defined as an ordered pair $\left(d^-_{G_1\star G_2}(x_1,x_2), d^+_{G_1\star G_2}(x_1,x_2)\right)$ where

$$d^-_{G_1\star G_2}(x_1,x_2) = \left(\sum_{\substack{(y_1,y_2)\in V_1\star V_2: \\ x_1=y_1 \in V_1,\, x_2y_2\notin E_2}} min\{M_{\sigma_1 L}(x_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\} + \sum_{\substack{(y_1,y_2)\in V_1\star V_2: \\ x_1y_1\in E_1, x_2y_2\in E_2}} min\{M_{\mu_1 L}(x_1,y_1), M_{\mu_2 L}(x_2,y_2)\}\right) - \left(\sum_{\substack{(y_1,y_2)\in V_1\star V_2: \\ x_1=y_1 \in V_1,\, x_2y_2\notin E_2}} max\{N_{\sigma_1 L}(x_1), N_{\sigma_2 L}(x_2), N_{\sigma_2 L}(y_2)\} + \sum_{\substack{(y_1,y_2)\in V_1\star V_2: \\ x_1y_1\in E_1,\, x_2y_2\in E_2}} max\{N_{\mu_1 L}(x_1,y_1), N_{\mu_2 L}(x_2,y_2)\}\right)$$

and

$$d^+_{G_1\star G_2}(x_1,x_2) = \left(\sum_{\substack{(y_1,y_2)\in V_1\star V_2: \\ x_1=y_1 \in V_1,\, x_2y_2\notin E_2}} min\{M_{\sigma_1 U}(x_1), M_{\sigma_2 U}(x_2), M_{\sigma_2 U}(y_2)\} + \sum_{\substack{(y_1,y_2)\in V_1\star V_2: \\ x_1y_1\in E_1, x_2y_2\in E_2}} min\{M_{\mu_1 U}(x_1,y_1), M_{\mu_2 U}(x_2,y_2)\}\right) - \left(\sum_{\substack{(y_1,y_2)\in V_1\star V_2: \\ x_1=y_1 \in V_1,\, x_2y_2\notin E_2}} max\{N_{\sigma_1 U}(x_1), N_{\sigma_2 U}(x_2), N_{\sigma_2 U}(y_2)\} + \sum_{\substack{(y_1,y_2)\in V_1\star V_2: \\ x_1y_1\in E_1, x_2y_2\in E_2}} max\{N_{\mu_1 U}(x_1,y_1), N_{\mu_2 U}(x_2,y_2)\}\right)$$

V. CONCLUSION

In this paper, boxdot product and star product on interval valued intuitionistic fuzzy graphs and some properties of these products on strong interval valued intuitionistic fuzzy graphs are discussed. We have determined the degree of vertices of these product graphs under certain conditions. Our future plan is to find some other properties of these product graphs.

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