15U601 (Pages: 2) Name: …………..……….

Reg. No………….………

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2018

(CUCBCSS-UG)

CC15U MAT6 B09 - REAL ANALYSIS

Mathematics - Core Course

(2015 Admission)

Time: Three Hours Maximum: 120 Marks

Section A

Answer ***all*** questions. Each question carries 1 mark

1. Give an example of a function which has absolute maximum and absolute minimum at every point of $\mathbb{R}$.
2. State true or false: ‘Continuous functions are always bounded’.
3. Does $4\sin x = x$ has a positive solution in $\left[\frac{\pi}{2}, \pi\right]$.
4. Find the norm of the partition P: (0,2,3,4).
5. State true or false: ‘An unbounded function cannot be Riemann integrable’.
6. Define primitive of f.
7. Evaluate $\lim \frac{nx}{1+(nx)^2}$.
8. Define uniform norm of a bounded function.
9. Is $f_n(x) = \frac{1}{x+n}$ is uniformly convergent in [0,b]; b > 0 ?
10. Give an example of an improper integral.
11. Define Beta function.
12. Find the value of $\Gamma(3)$.

(12 x 1 = 12Marks)

Section B

Answer any ***ten*** questions. Each question carries 4 marks

13. Let I be a closed bounded interval and let $f: I \to \mathbb{R}$ be continuous on I. Prove that f(I) is a closed bounded interval.
14. Define Lipschitz function. Prove that a Lipschitz function $f: A \to \mathbb{R}$ is uniformly continuous on A.
15. State maximum minimum theorem. Find the absolute maximum and absolute minimum of $g(x) = x^2$ in A = [-1,1].
16. Define Riemann integral of a function. Show that a constant function defined on [a, b] is Riemann integrable on [a, b].
17. Use fundamental theorem of calculus to evaluate $\int_a^b x\,dx$.
18. If $f(x) < g(x)\ \forall\ x \in [a, b]$, then show that $\int_a^b f(x)dx < \int_a^b g(x)\,dx$.

19. Check the uniform convergence of $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots$ in [-1,1].

20. Show that $G(x) = x^n(1 - x), x \in [0,1]$ converges uniformly on [0,1].

21. If (f_n) is a sequence of continuous functions converging uniformly to f, prove that f is continuous.

22. Define conditional convergence with a suitable example.

23. Express $\int_0^1 \frac{x^2}{\sqrt{1-x^5}} dx$ as a Beta function.

24. Find $\int_0^2 (8 - x^3)^{\frac{-1}{3}} dx$.

25. Show that $B(1, n) = \frac{1}{n}$.

26. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(10 x 4 = 40 Marks)

Section C

Answer any ***six*** questions. Each question carries 7 marks

27. If f and g are uniform continuous on a subset A of $\mathbb{R}$, then prove that f + g is uniformly continuous on A.

28. Show that $f(x) = x(x - 1)(x - 2)(x - 3)$ has four roots in [0,5].

29. If $f \in R[a, b]$ then prove that f is bounded on [a,b].

30. State and prove product theorem on Riemann Integration.

31. State and prove Weierstrass M test for the series.

32. If $f: A \to \mathbb{R}$is uniformly continuous on $A \subseteq \mathbb{R}$ and (x_n) is a Cauchy sequence in $\mathbb{R}$, prove that $f(x_n)$is a Cauchy sequence in $\mathbb{R}$.

33. Define Cauchy principal value of $\int_{-\infty}^{\infty} f(x)\, dx$. Also evaluate it for $\int_{-\infty}^{\infty} \sin x\, dx$.

34. Show that $B(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$.

35. Prove that $\int_0^{\infty} e^{-ax} x^{n-1} dx = \frac{\Gamma n}{a^n}$.

(6 x 7 = 42 Marks)

Section D

Answer any ***two*** questions. Each question carries 13 marks

36. (a) State and prove intermediate value theorem.

    (b) Prove that $\sin x$ is uniformly continuous on $[0, \infty]$.

37. (a) State and prove Cauchy criterion for uniform convergence of a sequence of functions.

    (b) Test for uniform convergence $\sum_1^{\infty} \frac{\sin nx}{n^p}$ for $p > 1$.

38. (a) Let $f: [a, b] \to \mathbb{R}$ and $c \in (a, b)$. Prove that $f \in R[a, b]$ if and only if its restrictions to [a,c] and [c,b] are both Riemann integrable and $\int_a^b f = \int_a^c f + \int_c^b f$.

    (b) Evaluate $\int_{-10}^{10} \text{Sgn}\,(x)\, dx$ where Sgn (x) denotes Signum function on [-10,10].

(2 x 13 = 26 Marks)

****