

FDTD Analysis of the Interaction of a Gaussian Pulse with Negative Permittivity Metamaterial Slab

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Abstract—The interaction of a Gaussian pulse with a negative permittivity metamaterial slab is presented. The results of this numerical study predict the propagation behavior of electromagnetic wave in presence of the slab. To perform this analysis the Auxiliary Differential Equation Finite Difference Time Domain (ADE-FDTD) analysis for a dispersive medium using Drude model is used. The negative permittivity plasma medium is modeled using effective medium theory. A Gaussian pulse of wide frequency bandwidth is used for the analysis. The field distributions for frequencies above and below the plasma frequency are investigated. The study reveals the frequency selective properties of the negative permittivity slab in relation to the reflection and transmission characteristics.

Index Terms—Dispersive medium, Drude model, FDTD, Gaussian pulse, Metamaterials, Negative Permittivity.

I. INTRODUCTION

Metamaterials are engineered periodic structures having homogeneous properties for the interacting waves. These materials show unusual electromagnetic properties not observed with normal materials. The unusual behaviors of these materials are due to its negative values of permeability μ , permittivity ϵ and index of refraction n . Periodic structures having negative μ and ϵ are mixed together to get negative n structure [1]–[4]. Finite Difference Time Domain (FDTD) method offers a simple and straight forward way for modeling complicated periodic structures [5]–[9] like metamaterial and photonic band-gap structures. Since these artificial plasma media consists of periodic distribution of conducting elements, it is dispersive [10]–[13]. There are two possibilities for the modeling of this medium in FDTD. One is the actual insertion of conducting elements periodically in the computational domain by assigning high electron density for the selected cells and the other is by considering the medium in terms of effective medium properties. Mainly three methods are used for the analysis of a dispersive medium using FDTD [5]. They are Auxiliary differential Equation method (ADE), Recursive Convolution method (RC), and the Z-transform method.

The electromagnetic wave propagation in presence of a negative permittivity slab using effective medium theory is preformed in this study. A Gaussian pulse of wide frequency bandwidth is used to illuminate the computational domain. The signal strengths in front and back of the negative permittivity slab for different frequencies are analyzed.

II. FORMULATION OF THE PROBLEM

The problem is simulated in 2-dimensional dispersive FDTD space using effective medium theory. The model equations involved in 2D-ADE-FDTD are used. In this study Drude model is used for modeling frequency dependent permittivity slab. According to this model, the relative permittivity $\epsilon_r(\omega)$ and relative permeability $\mu_r(\omega)$ are as follows,

$$\epsilon_r(\omega) = 1 - \frac{\omega_{ep}^2}{\omega^2 - \omega_{e0}^2 - i\gamma_e\omega} \quad (1)$$

$$\mu_r(\omega) = 1 - \frac{\omega_{mp}^2}{\omega^2 - \omega_{m0}^2 - i\gamma_m\omega} \quad (2)$$

where ω_{ep} is the electric plasma frequency and ω_{e0} is the low frequency edge of the electric forbidden band. γ_e is the electric damping factor. ω_{mp} , ω_{m0} and γ_m are the corresponding factors for the magnetic field. For a TE wave with field components E_z , H_x and H_y , Maxwells curl equations in component form are given by,

$$\frac{\partial D_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \quad (3)$$

$$\frac{\partial B_x}{\partial t} = -\frac{\partial E_z}{\partial y} \quad (4)$$

and

$$\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x} \quad (5)$$

The expressions for D and B in terms of the components of electric and magnetic fields E_z , H_x and H_y can be found out using Maxwells equations and Drude model as [7],

$$D_z = \epsilon_0 \left(1 - \frac{\omega_{ep}^2}{\omega^2 - \omega_{e0}^2 - i\gamma_e\omega}\right) E_z \quad (6)$$

$$B_x = \mu_0 \left(1 - \frac{\omega_{mp}^2}{\omega^2 - \omega_{m0}^2 - i\gamma_m\omega}\right) H_x \quad (7)$$

$$B_y = \mu_0 \left(1 - \frac{\omega_{mp}^2}{\omega^2 - \omega_{m0}^2 - i\gamma_m\omega}\right) H_y \quad (8)$$

Equations used for modeling of metamaterials are obtained by simplifying and converting to time domain by replacing $i\omega$ by $\partial/\partial t$ and applying the second order FDTD discretization to the Eqn. 6 - Eqn. 8.

The computational domain is divided into three regions and a negative permittivity slab of thickness d is defined in it as