33. A Company has 5 jobs to be done. The following matrix shows the return in rupees on assigning $i^{t h}(i=1,2,3,4,5)$ machine to the $j^{t h}(j=A, B, C, D, E)$ job. Assign the five jobs to the five machines so as to maximize the total expected profit.

| Machines | Jobs |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D | E |  |
| 1 | 5 | 11 | 10 | 12 | 4 |  |
| 2 | 2 | 4 | 6 | 3 | 5 |  |
| 3 | 3 | 12 | 5 | 14 | 6 |  |
| 4 | 6 | 14 | 4 | 11 | 7 |  |
| 5 | 7 | 9 | 8 | 12 | 5 |  |

## Section D

Answer any two questions. Each question carries 10 marks.
34. Use Simplex Method to Solve the LPP.

Maximize $z=2 x_{1}+3 x_{2}$
Subject to the constrains: $-x_{1}+2 x_{2} \leq 4$

$$
x_{1}+x_{2} \leq 6
$$

$$
x_{1}+3 x_{2} \leq 9
$$

$$
x_{1}, x_{2} \text { unrestricted }
$$

35. Use two-phase simplex method to solve the LPP

Maximize $z=2 x_{1}+x_{2}-x_{3}$
Subject to the constrains: $4 x_{1}+6 x_{2}+3 x_{3} \leq 8$

$$
\begin{array}{r}
3 x_{1}-6 x_{2}-4 x_{3} \leq 1 \\
2 x_{1}+3 x_{2}-5 x_{3} \geq 4 \\
x_{1}, x_{2}, x_{3} \geq 0
\end{array}
$$

36. Find the optimum solution of the following transportation problem whose unit cost matrix is given as under:

| From | To |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | I | II | III | IV | Supply |
| A | 7 | 10 | 14 | 8 | 35 |
| B | 7 | 11 | 12 | 6 | 40 |
| C | 5 | 8 | 15 | 9 | 25 |

Name:
Reg. No... $\qquad$

## SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2019

(Regular/Supplementary/Improvement)
(CUCBCSS-UG)

## CC15U MAT6 E02 - LINEAR PROGRAMMING

Mathematics - Elective
(2015 Admission onwards)
Time: Three Hours
Maximum: 80 Marks

## Section A

Answer all questions. Each question carries 1 mark.

1. Define a Convex set in $\mathbb{R}^{n}$
2. If $A=\{1,2\}$ then $\langle A\rangle=\cdots$
3. State fundamental theorem of linear programming.
4. Give a necessary and sufficient condition for a basic feasible solution to an L. P.P.
5. When the basic solution to $\boldsymbol{A} \boldsymbol{x}=\boldsymbol{b}$ is said to be degenerate?
6. Define Artificial variable.
7. When does Big-M method indicate that the L.P.P. has no solution?
8. In the two-phase simplex method, what is the objective of Phase-1
9. If the $i^{\text {th }}$ constraints of the primal problem is an equality then the $i^{\text {th }}$ dual variable is...
10. A necessary and sufficient condition for the existence of a feasible solution to the transportation problem is ...
11. A feasible solution to a transportation problem is basic, if and only if, the corresponding cells in the transportation table do not contain ...
12. Define a loop in transportation problem.

## Section B

Answer any nine questions. Each question carries 2 marks.
13. A factory uses three different resources for the manufacture of two different products, 20 units of resource $A, 12$ units of $B$ and 16 units of $C$ being available. 1 units of the first product requires 2,2 and 4 units of the respective resources and 1 units of the second requires 4,2 , and 0 units of the respective resources. It is known that the first product gives a profit of Rs. 2 per unit and the second 3. Formulate the linear programming problem for maximizing the profit.
14. Prove that the closed half spaces $H=\{x: c . x \geq z\}$ is a convex set.
15. Let $S \subset \mathbb{R}^{n}$ be a convex set. Prove that Int $S$ is also convex.
16. Verify Minimax theorem for the function $f(x)=\{9,7,5,3,1\}$
17. Find a basic solution to the system of linear equations

$$
\begin{gathered}
x_{1}+2 x_{2}+x_{3}=4 \\
2 x_{1}+x_{2}+5 x_{3}=5
\end{gathered}
$$

18. Reduce the following L.P.P. in standard form

Minimize $z=12 x_{1}+5 x_{2}$
Subject to the constraints: $6 x_{1}+3 x_{2} \geq 15$

$$
\begin{aligned}
7 x_{1}+2 x_{2} & \leq 14 \\
x_{1}, x_{2} & \geq 0
\end{aligned}
$$

19. Solve

Minimize $z=3 x-2 y$
Subject to the constraints: $x-y \leq 1$,

$$
\begin{array}{r}
3 x-2 y \leq 6 \\
x, y \geq 0
\end{array}
$$

20. How do we solve an unbalanced transportation problem?
21. What is the difference between feasible solution and basic feasible solution?
22. Obtain an initial basic feasible solution to the following transportation problem using North-

West corner rule

| Origin | Destination |  |  |  | Available |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | D1 | D2 | D3 | D4 |  |
| O1 | 6 | 4 | 1 | 5 | 14 |
| O2 | 8 | 9 | 2 | 7 | 16 |
| O3 | 4 | 3 | 6 | 2 | 5 |
| Demand | 6 | 10 | 15 | 4 |  |
|  |  |  |  |  |  |

23. How do we solve assignment problem when it has restrictions on assignments?
24. Explain the difference between a transportation problem and assignment problem.

## (9) 2 = 18 Marks)

## Section C

Answer any six questions. Each question carries 5 marks
25. Solve the following linear programming problem by graphical method

Minimize $z=2 x_{1}+x_{2}$
Subject to the constraints: $5 x_{1}+10 x_{2} \leq 50$

$$
\begin{aligned}
x_{1}+x_{2} & \geq 1 \\
x_{2} & \leq 4 \\
x_{1}, x_{2} & \geq 0
\end{aligned}
$$

26. Prove that the set of all convex combinations of a finite number of vectors $x_{1}, x_{2}, \ldots, x_{k}$ in $\mathbb{R}^{\boldsymbol{n}}$, is a convex set.
27. For any points $x, y$ in $\mathbb{R}^{n}$, show that the line segment $[x: y]$ is a convex set.
28. Prove that the set of feasible solutions to an L.P.P is a convex set.
29. Verify that the dual of dual is a primal for the following LPP

Maximize $z=8 x_{1}+3 x_{2}$
Subject to the constraints

$$
\begin{aligned}
x_{1}-6 x_{2} & \leq 2 \\
5 x_{1}+7 x_{2} & =-4 \\
x_{1}, x_{2} & \geq 0
\end{aligned}
$$

30. Solve the following cost-minimizing assignment problem:

| Machines | Jobs |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D |
| 1 | 18 | 26 | 17 | 11 |
| 2 | 13 | 28 | 14 | 26 |
| 3 | 38 | 19 | 18 | 15 |
| 4 | 19 | 26 | 24 | 10 |

31. Use Big-M method to solve

Maximize $z=3 x_{1}+2 x_{2}+3 x_{3}$
Subject to the constraints

$$
\begin{gathered}
2 x_{1}+x_{2}+x_{3} \leq 2 \\
3 x_{1}+4 x_{2}+2 x_{3} \geq 8 \\
x_{1}, x_{2} x_{3} \geq 0
\end{gathered}
$$

32. Obtain an initial basic feasible solution to the following transportation problem using Vogel's Approximation method

| Origin | Destination |  |  | Available |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ |  |  |  |  |
| $\mathrm{O}_{1}$ | 18 | 26 | 17 | 10 |  |  |  |
| $\mathrm{O}_{2}$ | 13 | 28 | 14 | 12 |  |  |  |
| $\mathrm{O}_{3}$ | 38 | 19 | 18 | 14 |  |  |  |
| $\mathrm{O}_{4}$ | 19 | 26 | 24 | 9 |  |  |  |
| Demand | 14 | 8 | 23 |  |  |  |  |
|  |  |  |  |  |  |  |  |

