20U108 (Pages: 2) Name: ………………………..

Reg. No……..……………….

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CUCBCSS-UG)

CC19U MTS1 B01 - BASIC LOGIC AND NUMBER THEORY

(Mathematics - Core Course)

(2019 Admission – Supplementary/Improvement)

Time: 2½ Hours Maximum: 80 Marks

Credit: 4

PART A

Answer ***all*** questions. Each question carries 2 marks.

1. Find the truth value of the compound statement (5 < 8) and (2 + 3 = 4).
2. State the Idempotent laws of logic and construct the truth table.
3. Rewrite the sentence "Some chalkboards are black" symbolically.
4. Explain vacuous proof.
5. Prove that there is no positive integer between 0 and 1.
6. Find the quotient and the remainder when -325 is divided by 13.
7. Express 3014 in base eight.
8. Prove that 2 consecutive Fibonacci numbers are relatively prime.
9. Find LCM of 1050 and 2574 using canonical decomposition.
10. Does every linear Diophantine equation have a solution? Justify.
11. Define congruence modulo m. Rewrite the given sentence using the congruence symbol: n is an even integer.
12. Give counter example to disprove the statement: If $a^2 \equiv b^2 \pmod{m}$ then $a \equiv b \pmod{m}$.
13. If n is an even integer, then prove that $\varphi\ (2n) = 2\ \varphi(n)$.
14. Let p be a prime and k be any positive integer, then what is $\varphi\ (p^k)$.
15. Find the number and sum of divisors of 180.

(Ceiling: 25 Marks)

PART B

Answer ***all*** questions. Each question carries 5 marks.

16. Test the validity of the argument:

$$p \leftrightarrow q$$
$$\sim p \vee r$$
$$\sim r$$
$$\ldots\ldots\ldots\ldots$$
$$\text{Therefore } \sim q$$

17. Prove by contradiction that there is no largest prime number.

18. (a) Let a and b be any positive integers. Then prove that the number of positive integers less than or equal to a and divisible by b is $\left\lfloor \frac{a}{b} \right\rfloor$

(b) Find the number of positive integers in the range 1976 through 3776 that are not divisible by 17.

19. Write the definition of Fermat numbers. Let f_n denote the n^{th} Fermat number. Write the recursive definition of f_n. Prove that every prime factor of f_n is of the form $k2^{n+2} + 1, n \geq 2$. Hence show that $f_4 = 65537$ is prime.

20. Express the gcd of 14 and 18 as linear combination of the same numbers.

21. Using inverses, find the incongruent solutions of the linear congruence $4x \equiv 11 \pmod{13}$.

22. Show that for any integer n, the number $n^{33} - n$ is divisible by 15.

23. For $n \geq 1$, prove that the sum of the positive integers less than n and relatively prime to n is $\frac{1}{2} n\, \varphi(n)$.

(Ceiling: 35 Marks)

PART C

Answer any ***two*** questions. Each question carries 10 marks.

24. Discuss various methods of proofs for proving theorems.

25. Let a and b be any positive integers and r is the remainder when a is divided by b. Then prove that (a, b) = (b, r). Hence evaluate (2076, 1776).

26. Solve the LDE $x + 2y + 3z = 6$ and $2x - 3y + 4z = 5$

27. (a) State and prove Wilson's theorem.

(b) Prove that if $(n-1)! = -1 \ (mod\ n)$, then n is a prime.

(2 x 10 = 20 Marks)

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