20P104 (Pages: 2) Name: ……………………….

Reg. No……………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C04 - DISCRETE MATHEMATICS

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define a lattice. Give an example.
2. Let $(X, +, \ . \ , \ ')$ be a Boolean algebra. Prove that $(x')' = x$ for all $x \in X$.
3. Find all the characteristic numbers of the function $f(x_1, x_2, x_3) = x_1x_2 + x_2x_3 + x_3x_1$.
4. Show that the sequence $d = (7,6,3,3,2,1,1,1)$ is not graphical.
5. Show that if G is a self-complementary graph of order n, then $n \equiv 0$ or $1 \ (mod\ 4)$.
6. Prove that the Petersen graph is nonplanar.
7. Find a grammar for $\Sigma = \{a, b\}$ that generates the set of all strings with at least one a.
8. If $\Sigma = \{0,1\}$, design an *nfa* to accept the set of strings either ending with two consecutive zeros or two consecutive ones.

(8 × 1 = 8 weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT- I

9. Let X be a finite set and $\leq$ be a partial order on X. R is a binary relation on X defined by xRy iff y covers x. Prove that $\leq$ is the smallest order relation containing R.
10. Let $(X, +, \ . \ , \ ')$ be a finite Boolean algebra. Prove that every non-zero element of X contains at least one atom.
11. Write the Boolean function $f(a, b, c) = a + b + c'$, in their disjunctive normal form.

UNIT- II

12. Prove that for any loopless connected graph $G, \kappa(G) \leq \lambda(G) \leq \delta(G)$.
13. Prove that a graph is planar if and only if it is embeddable on a sphere.
14. Let G be a planar graph with at least three vertices, then show that $m \leq 3n - 6$.

UNIT- III

15. (a) Prove that $(w^R)^R = w$ for all $w \in \Sigma^*$.

(b) Find the grammar for the language $L = \{w: |w| \bmod 3 = 0\}$ on $\Sigma = \{a\}$.

16. Find a *dfa* for the language $L = \{w: (n_a(w) - n_b(w)) \bmod 3 > 0\}$ on $\Sigma = \{a, b\}$.

17. Design an *nfa* with no more than 5 states for the set $\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$

(6 × 2 = 12 weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Prove that $(X, \leq)$ is a lattice where $(X, +, \cdot, ')$ is the Boolean algebra and $\leq$ is defined in X by $x \leq y$ if and only if $x.y' = 0$. Find the maximum and minimum elements of this lattice.

(b) Prove that every finite Boolean algebra is isomorphic to a power set Boolean algebra.

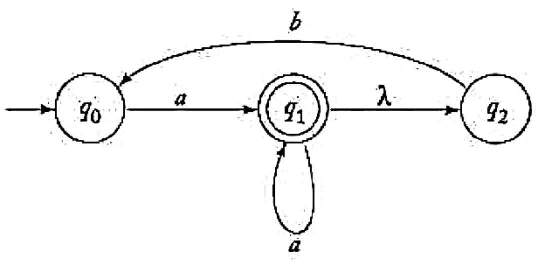
19. Prove that the set of all symmetric Boolean functions of n Boolean variables x_1, $x_2, \ldots, x_n$ is a subalgebra of the Boolean algebra of all Boolean functions of these variables. Also prove that as a Boolean algebra, it is isomorphic to the power set Boolean algebra of the set $\{1, 2, \ldots, n\}$.

20. (a) Prove that a graph is bipartite if and only if it contains no odd cycles.

(b) Prove that a simple graph is a tree if and only if any two distinct vertices are connected by a unique path.

21. (a) Show that the language $L = \{awa : w \in \{a, b\}^*\}$ is regular.

(b) Find a *dfa* equivalent to the following *nfa*.



(2 × 5 = 10 weightage)

****