19P259 (Pages: 2) Name…………………………

Reg. No.........………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCSS - PG)

CC19P MST2 C07 - ESTIMATION THEORY

(Statistics)

(2019 Admissions - Regular)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Define (i) minimal sufficient statistic and (ii) Ancillary statistics. Illustrate both with an example for each.
2. Define an unbiased estimator. Do the unbiased estimators always exist? Justify your claim.
3. Describe percentile method for finding consistent estimator.
4. Find the moment estimator of the parameters of Gamma distribution.
5. Define the terms (i) Prior distribution (ii) Posterior distribution and (iii) Loss functions.
6. Explain the construction of Bayesian confidence interval.
7. Define (i) shortest expected length confidence interval and (ii) fiducial interval.

(4 x 2 = 8 Weightage)

Part B

Answer any ***four*** questions. Each carries 3 weightage.

8. State and prove Rao Blackwell theorem. Explain its application through an example.
9. Define Cramer Rao inequality. Obtain the Cramer Rao lower bound for the variance of the unbiased estimator of μ, when a r.s of size n is taken from $N(\mu, \sigma^2)$.
10. Define a consistent estimator. Explain the criteria to choose between two consistent estimators. Illustrate with an example.
11. Define a CAN estimator. Establish the invariance property of CAN estimators.
12. Obtain the maximum likelihood estimator (m.l.e.) of the parameter θ when a random sample of size n is taken from: (i) $f(x, \theta) = \theta x^{\theta-1}; 0 < x < 1$ *and* α *known.*
    (ii) $X(\theta - 1, \theta + 1)$)
13. Let $x_1, x_2, \dots, x_n$ is a random sample from Poisson (λ). Assuming the prior distribution of λ as exponentially distributed with mean 1, obtain the Bayes estimate of λ.

14. Describe the construction of $100(1-\alpha)\%$ confidence interval for the mean of normal population with (i) known variance and (ii) unknown variance.

(4 x 3 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

15. (a) If X_1, X_2 are random samples of size 2 from Poisson (λ), show that $X_1 + X_2$ is sufficient but $X_1 + 2X_2$ is not sufficient.

    (b) Distinguish between UMVUE and MVB estimator. Construct MVB estimator for the parameter θ when a random sample is taken from the family with probability density function $f(x) = \begin{cases} \theta e^{-\theta x} & when\ x > 0 \\ 0 & otherwise \end{cases}$

16. (a) Prove or disprove : If t is a consistent estimator of θ , then t^2 is a consistent estimator of θ^2.

    (b) If n independent observations $X_1, X_2, \dots, X_n$ are drawn from the Poisson distribution with parameter λ, discuss the unbiasedness and consistency of the estimator $T = \frac{2}{n(n+1)} \sum_{i=1}^{n} i\, X_i$ for λ.

17. (a) Show that under some regularity conditions the maximum likelihood estimator is consistent and asymptotically normal.

18. (a) Let $x_1, x_2, \dots, x_n$ is a random sample from $U(0, \theta)$. Obtain an unbiased confidence interval for θ.

    (b) Describe the construction of $100(1-\alpha)\%$ large sample confidence interval for the variance of Normal population with unknown mean.

(2 x 5 = 10 Weightage)

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