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## SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCSS - PG)
CC19P MTH2 C09 - OPERATIONS RESEARCH
(Mathematics)
(2019 Admissions - Regular)
Time: Three Hours
Maximum: 30 Weightage

## PART A

Answer all questions. Each question carries 1 weightage.

1. Show that the sum of two convex function is a convex function.
2. Define artificial variables.
3. Define basic feasible solution?
4. Explain the procedure when the cost was changed in sensitivity analysis.
5. Explain degeneracy in Transportation Problem.
6. Write general form of mixed integer linear programming problem.
7. Define spanning tree of a graph. Give example?
8. Explain dominance property in game theory?
( $8 \times 1=8$ Weightage)

## PART B

Two questions should be answered from each unit. Each question carries 2 weightage.

## UNIT I

9. Let $f(X)$ be a convex differentiable function defined in a convex domain $K \subseteq E_{n}$. Then prove that $f\left(X_{0}\right), X_{0} \in K$ is a global minimum if and only if $\left(X-X_{0}\right)^{\prime} \nabla f\left(X_{0}\right) \geq 0$ for all $X$ in $K$
10. If $f(x)$ is minimum at more than one vertices of $S_{F}$ then it is minimum at all those points which are the convex linear combination of these vertices.
11. Solve by big M method: Minimize $f(x)=4 x_{1}+5 x_{2}$

Subject to

$$
\begin{aligned}
2 x_{1}+x_{2} & \leq 6 \\
x_{1}+2 x_{2} & \leq 5 \\
x_{1}+x_{2} & \geq 1, \\
x_{1}+4 x_{2} & \geq 2, \\
x_{1} \geq 0, x_{2} & \geq 0
\end{aligned}
$$

## UNIT II

12. Prove that the transportation problem has a triangular basis.
13. Solve the following for minimum cost starting with the degenerate solution $x_{12}=30, x_{21}=40, x_{32}=20, x_{43}=60$

|  | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ |  |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{O}_{1}$ | 4 | 5 | 2 | 30 |
| $\mathrm{O}_{2}$ | 4 | 1 | 3 | 40 |
| $\mathrm{O}_{3}$ | 3 | 6 | 2 | 20 |
| $\mathrm{O}_{4}$ | 2 | 3 | 7 | 60 |
|  | 40 | 50 | 60 |  |

14. Write the dual of Minimize $f(X)=6 x_{1}+3 x_{2}$

Subject to

$$
\begin{aligned}
& 3 x_{1}+4 x_{2}+x_{3} \geq 5 \\
& 6 x_{1}-3 x_{2}+x_{3} \geq 2 \\
& x_{1} \geq 0, x_{2} \geq 0, x_{3} \geq 0
\end{aligned}
$$

## UNIT III

15. Prove that the maximum flow in a graph is equal to the minimum of the capacities of all possible cuts in it.
16. Prove the algorithm for minimum spanning tree.
17. Explain Rectangular game as an LP problem.

## PART C

Answer any two questions. Each question carries 5 weightage.
18. Solve the problem: using simplex method Maximize $\mathrm{f}=-5 x_{1}+13 x_{2}+5 x_{3}$

Subject to

$$
\begin{gathered}
12 x_{1}+10 x_{2}+4 x_{3} \leq 90 \\
-x_{1}+3 x_{2}+x_{3} \leq 20 \\
x_{1} \geq 0, \quad x_{2} \geq 0, \quad x_{3} \geq 0
\end{gathered}
$$

Find the change in optimal solution when right side of the second constraint is changed to 30 using sensitivity analysis.
19. Explain branch and bound method?
20. Prove that a vertex $S_{F}$ has a feasible solution.
21. Solve graphically the game with payoff matrix $\left[\begin{array}{cc}2 & 7 \\ 3 & 5 \\ 11 & 2\end{array}\right]$

