## FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCSS-PG)

Mathematics

## CC17P MT4 E10 / CC18P MT4 E10 - ADVANCED OPERATIONS RESEARCH

(2017 Admission onwards)

Time: Three Hours

Maximum: 36 weightage

## PART A

Answer all questions. Each question carries 1 weightage.

- 1. What do you mean by separable programming?
- 2. State the Kuhn-Tucker theorem.
- 3. Solve graphically:

Minimize  $f = (x_1 - 2)^2 + x_2^2$ 

subject to  $x_1^2 + x_2 - 1 \le 0$ ,

$$x_1 \ge 0, x_2 \ge 0.$$

- 4. Write the general form of a Quadratic programming problem.
- 5. Explain the term posynomial with a suitable example.
- 6. Convert the following problem into the form of a geometric programming problem:

Find the dimensions of a rectangle of maximum area inscribed in a circle of radius r.

- 7. Write the standard form of a geometric programming problem.
- 8. Explain the primal-dual concept in geometric programming.
- 9. What is serial multistage model in dynamic programming?
- 10. Define the term forward recursion used in dynamic programming.
- 11. State Bellman's principle of optimality.
- 12. Explain the constraint with negative terms in geometric programming problem.
- 13. Define the term decision variables and state variables in a dynamic programming problem.
- 14. Define decomposability in an optimization problem.

### (14 x 1 = 14 Weightage)

# PART B

Answer any seven questions. Each question carries 2 weightage.

- 15. Describe how a non linear function can be approximated in a domain by a piecewise linear function.
- 16. Write the orthogonality conditions in a general geometric programming problem.
- 17. Discuss how the geometric programming problem can be generalized through Kuhn Tucker Theory.

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- 18. If  $X_0$  is a solution of the convex programming problem, Minimize  $f(X), X \in E_n$  subject to  $g_i(X) \le 0, i = 1, 2, ..., m, X \ge 0$  and if the set of points X such that G(X) < 0 is non empty, then prove that there exists a vector  $Y_0 \ge 0$  in  $E_m$  such that  $f(X) + Y'_0 G(X) \ge f(X_0)$ .
- 19. Minimize  $f(X) = (x_1 + 1)^2 + (x_2 2)^2$ subject to  $x_1 - 2 \le 0$ ,  $x_2 - 1 \le 0$ ,  $x_1, x_2 \ge 0$ .
- 20. Explain the terms weight functions and normalized weight functions in geometric programming problems.
- 21. Discuss the computational economy in dynamic programming.
- 22. Determine  $\max(u_1^2 + u_2^2 + u_3^2)$  subject to  $u_1u_2u_3 \le 6$  where  $u_1, u_2, u_3$  are positive integers.
- 23. Describe a method in dynamic programming to solve the problem: Minimize  $\sum_{j=1}^{n} f_j(u_j)$

subject to  $\sum_{j=1}^{n} a_j u_j \ge b$ 

$$u_i, a_i \ge 0, j = 1, 2, \dots, n, b > 0.$$

24. What are the essential features of dynamic programming problem?

(7 x 2 = 14 Weightage)

### PART C

Answer any two questions. Each question carries 4 weightage.

25. Solve by the method of quadratic programming:

*Minimize*  $-6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2$  subject to  $x_1 + x_2 \le 2$ ,  $x_1$ ,  $x_2 \ge 0$ .

26. Solve the geometric programming problem:

Minimize  $f(X) = \frac{c_1}{x_1 x_2 x_3} + c_2 x_2 x_3$  subject to  $g_1(X) = c_3 x_1 x_3 + c_4 x_1 x_2 = 1$  and  $c_i > 0, x_j > 0, i = 1, 2, 3, 4, j = 1, 2, 3.$ 

- 27. Maximize  $\sum_{n=1}^{4} (4u_n nu_n^2)$  subject to  $\sum_{n=1}^{4} u_n = 10, u_n \ge 0$ .
- 28. Prove that in a serial two stage minimization or maximization problem if (i) the objective function  $\phi_2$  is a separable function of stage returns  $f_1(X_1, U_1)$  and  $f_2(X_2, U_2)$ , and (ii)  $\phi_2$  is a monotonic non decreasing function of  $f_1$  for every feasible value of  $f_2$ , then the problem is decomposable.

#### $(2 \times 4 = 8 \text{ Weightage})$