17U601 (Pages: 2) Name: ……………………

Reg. No……………………

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCBCSS-UG)

(Regular/Supplementary/Improvement)

CC15U MAT6 B09 - REAL ANALYSIS

Mathematics - Core Course

(2015 Admission onwards)

Time: Three Hours Maximum: 120 Marks

SECTION A

Answer ***all*** questions. Each question carries 1 mark.

1. The cluster point of B = $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ is …………
2. The absolute maximum of $sin\ x$ is ……………
3. Give an example to show that a continuous function does not necessarily have an absolute maximum or absolute minimum on the set.
4. Give an example of a continuous function which is not uniformly continuous.
5. The norm of partition P $= (0, 1.5, 2, 3.4, 4)$ of $[0, 4]$ is…………
6. Give an upper bound for the error estimate in trapezoidal approximations.
7. If $f_n(x) = \log\left(\frac{x}{n}\right)$, show that $\lim_{n\to\infty}\left(f_n(x)\right) = 0$.
8. Find the radius of convergence of $\sum x^n$
9. Find the uniform norm of the function $f_n(x) = \frac{x}{n}$, where $x \in [0, 1]$
10. Give an example of improper integral of the first kind.
11. For n > 1, $\Gamma n =$………..
12. The CPV of $\int_{-\infty}^{\infty} x\ dx$ = ………..

(12 x 1 = 12 Marks)

SECTION B

Answer any ***ten*** questions. Each question carries 4 marks

13. Is there a real number that is one less than its fifth power?
14. Show that $g(x) = \frac{1}{x}$ is uniformly continuous on A = $\{x \in \mathbb{R} : 1 \leq x \leq 2\}$.
15. Show that if f and g are uniformly continuous on a subset A of $\mathbb{R}$, then $f + g$ is uniformly continuous on A.
16. Using Fundamental theorem evaluate $\int_a^b x\ dx$
17. If f and g are in R[a, b] and if $f(x) \leq g(x)$ for all $x \in [a, b]$, prove that $\int_a^b f \leq \int_a^b g$.
18. If $\varphi(x) = x + 1\ for\ x \in [0, 1]$ & rational and $\varphi(x) = 0\ for\ x \in [0, 1]$ & irrational, prove that $\varphi \notin R[0, 1]$
19. If g $\in$ R[a, b] and if $f(x) = g(x)$ except for a finite number of points in $[a, b]$, prove that f $\in$ R[a, b] and $\int_a^b f = \int_a^b g$.
20. If $\varphi: [a, b] \to \mathbb{R}$ takes only a finite number of distinct values, is φ a step function?
21. Show that $G_n(x) = x^n(1 - x)$ for $x \in [0, 1]$ converges uniformly to $g(x) = 0$.

22. Prove that sequence (f_n) of bounded functions on A$\subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.
23. Show that $\cos x + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \ldots$ converges uniformly. Also give the interval of uniform convergence.
24. Show that $\beta(m,n) = \beta(n,m)$.
25. Investigate the convergence of $\int_1^{\infty} \frac{sin^2 x}{x^2} dx$.
26. Evaluate $\int_0^{\frac{\pi}{2}} sin^7\theta cos^5\theta d\theta$.

(10 x 4 = 40 Marks)

SECTION C

Answer any six questions. Each question carries 7 marks.

27. If $f: I \to \mathbb{R}$ is continuous on I = [a.b],a closed and bounded interval, show that f is bounded on I.
28. If $f: A \to \mathbb{R}$ is uniformly continuous on a set A of $\mathbb{R}$ and if (x_n) is a Cauchy sequence in A, show that $(f(x_n))$ is a Cauchy sequence in $\mathbb{R}$. Using this, show that $f(x) = \frac{1}{x}$ is not uniformly continuous on (0,1).
29. Using Taylor's theorem, approximate e^{-x^2} by a polynomial and use it to find an approximate value of $\int_0^1 e^{-x^2} dx$.
30. Let g: [0,3]→ $\mathbb{R}$ be defined by $g(x) = 2$ for $0 \leq x \leq 1$ and $g(x) = 3$ for $1 < x \leq 3$. Find $\int_0^3$ g.
31. If f and g are continuous on $[\mathrm{a}, \mathrm{b}]$ and if $\int_a^b f = \int_a^b g$, prove that there exists $c \in [a,b]$ such that $f(c) = g(c)$
32. State and prove Cauchy criterion for the uniform convergence of sequence of functions.
33. Show that (f_n), where $f_n(x) = \frac{n}{x+n}$, $x \geq 0$ is uniformly convergent in the closed bounded interval $[0,m]$, whatever m be, but not in the interval $0 \leq x < \infty$.
34. Show that $\int_a^{\infty} \frac{1}{x^p} dx$, a > 0 converges if p $>$ 1 and diverges if p $\leq$ 1.
35. Show that if p and q are positive, then $\beta(p+1,q) + \beta(p,q+1) = \beta(p,q)$.

(6 x 7 = 42 Marks)

SECTION D

Answer any ***two*** questions. Each question carries 13 marks.

36. (i) Show that if $f: A \to \mathbb{R}$ is a Lipchitz function, then f is uniformly continuous.
    (ii) Is the converse true? Justify with a suitable example.
37. State and prove Cauchy criterion for integrability. Using this check the integrability of Dirichlet function.
38. (i) Prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(\mathrm{m+n})}$
    (ii) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(2 x 13 = 26 Marks)

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