37. Solve the system of equations by Gauss elimination method.

$x + 2y + z - w = -2,$

$x + 2y + 3z - 2w = -6,$

$2x + 3y - z + 2w = 7,$

$x + y + z + w = 2$.

38. Evaluate $\int_0^{\pi/2} \sqrt{\sin x}\, dx$ with $h = \frac{\pi}{12}$ using

i. Trapezoidal rule

ii. Simpson's $1/3$ rule

iii. Simpson's $3/8$ rule.

($2 \times 13 = 26$ Marks)

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17U603 (Pages: 4) Name: ……..…….

Reg. No……..……

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCBCSS-UG)

(Regular/Supplementary/Improvement)

CC15U MAT6 B11 - NUMERICAL METHODS

Mathematics - Core Course

(2015 Admission onwards)

Time: Three Hours Maximum: 120 Marks

Section A

Answer ***all*** questions. Each question carries **1** mark.

1. Set up a Newton Raphson iterative formula for the equation $x = 2\sin x$.
2. Define central difference operator and write a relation connecting δ and E.
3. Show that $hD = \log E$.
4. State Newton's forward interpolation formula.
5. Write Gauss forward formula.
6. Write the Lagrange's polynomial of second degree.
7. Find the characteristic equation of the matrix $\begin{bmatrix} 5 & 1 & -1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$
8. Define spectrum of a matrix.
9. Write the fourth order Runge-Kutta formula.
10. Write Adam Bashforth predictor formula.
11. Given $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$. This problem is

    i) a linear differential equation.

    ii) a homogenous differential equation.

    iii) an initial value problem.

    iv) a boundary value problem.
12. The 19th difference of a polynomial of degree 18 is ……..

($12 \times 1 = 12$ Marks)

Section B

Answer any ***ten*** questions. Each question carries 4 marks.

13. Solve $x^3 - 9x + 1 = 0$ for the root between 2 and 4 by bisection method.
14. Briefly explain the method of false position.

15. Show that $\mu = \sqrt{1 + \frac{\delta^2}{4}}$

16. For the following table, write $f(x)$ as a function of x using Newton's divided difference formula.

x	0	1	2	4
$f(x)$	1	1	2	5

17. If $y(1) = 4,\ y(3) = 12,\ y(4) = 19$ and $y(x) = 7,$ find x.

18. Find Lagrange's interpolation polynomial fitting the points $f(1) = -3,\ f(3) = 0,\ f(4) = 30$ and $f(6) = 132$.

19. The sales (in lakhs) in a particular shop for certain years are given in the table below. Estimate the sales for the year 2003 using Newton's backward difference interpolation formula.

Year	1996	1998	2000	2002	2004
Sales	40	43	48	52	57

20. Using trapezoidal rule, evaluate $\int_0^{-2}(t^3 + t)\,dt$.

21. Decompose the matrix $\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ in the LU form.

22. Solve the following system by Gauss Jordan method.

$2x - 3y + 4z = 7,$

$5x - 2y + 2z = 7,$

$6x - 3y + 10z = 23.$

23. Solve the IVP $\frac{dy}{dx} = 1 + xy,\ y(0) = 1$ using Taylor's series method. Find $y(0.1)$.

24. Find $y(0.1)$ correct to four decimal places by Runge-Kutta second order formula, if $\frac{dy}{dx} = y - x,\ y(0) = 2.$

25. The differential equation $\frac{dy}{dx} = 1 + y^2$ satisfies the following data:

x	0	0.2	0.4	0.6
y	0	0.2027	0.4228	0.6841

Compute $y(0.8)$ by Milne's method.

26. Write down the difference between Jacobi's method and Gauss-Seidel method.

($10 \times 4 = 40$ Marks)

Section C

Answer any ***six*** questions. Each question carries 7 marks.

27. Find the smallest root of the equation $x^3 - 6x^2 + 11x - 6 = 0$.

28. Find the root lying between 5 and 8 of the equation $x^{2.2} = 69$ using secant method.

29. Using the method of separation of symbols,

show that $\Delta^n u_{x-n} = u_x - nu_{x-1} + \frac{n(n-1)}{2}u_{x-2} + \cdots + (-1)^n u_{x-n}$.

30. Find the value of $e^{1.17}$ using Gauss' forward formula from the table below:

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
e^x	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

31. Solve the following system by LU decomposition.

$10x + y + z = 12, \qquad 2x + 10y + z = 13, \qquad 2x + 2y - 10z = 14.$

32. Using Gauss Jordan method find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$.

33. Determine the largest eigen value and the corresponding eigen vector of the matrix

$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

34. Use Euler's method with $h = 0.1$ to solve the IVP $\frac{dy}{dx} = x^2 + y^2,\ y(0) = 0$ in the range $0 \le x \le 0.2$.

35. Solve $\frac{dy}{dx} = \frac{y-x}{y+x},\ y(0) = 1$ using Picard's method. Find $y(0.1)$.

($6 \times 7 = 42$ Marks)

Section D

Answer any ***two*** questions. Each question carries 13 marks.

36. A slider in a machine moves along a fixed straight rod. Its distance x cm along the rod is given below for various values of time t seconds. Find the velocity of the slider and its acceleration at time $t = 0.2$ seconds.

t	0.0	0.1	0.2	0.3	0.4	0.5	0.6
x	3.013	3.162	3.287	3.364	3.395	3.381	3.324