20U301 (Pages: 2) Name:...........................

Reg. No: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS3 B03 - CALCULUS OF SINGLE VARIABLE - 2

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit: 4

Part A (Short answer questions)

Answer all questions. Each question carries 2 marks.

1. Show that the functions $f(x)=e^{2x}$ and $g(x)=\ln \sqrt{x}$ are inverses of each other.
2. Evaluate ${\displaystyle\int 2^{-x} }dx$.
3. Find the derivative of $f(x)=(x+\cos x)^{\sqrt{2}}$.
4. Evaluate $\displaystyle\int_{-1}^{\infty}e^{-x}dx$
5. List the terms of the sequence $\left\{\dfrac{n+1}{2n-1}\right\}$
6. Determine whether the series $\displaystyle\sum_{n=1}^{\infty}3\left(\frac{-1}{2}\right)^{n-1}=3-\frac{3}{2}+\frac{3}{4}-\frac{3}{8}+\cdots$ converges or diverges.
7. Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{4}}$ converges or diverges.
8. State root test for series.
9. Find the radius of convergence and the interval of convergence of the power series $\sum \limits _{n=1}^{\infty}(nx)^n.$
10. Describe the curves represented by the parametric equations $x= \cos\theta +1$ and $y= \sin\theta -2$, with parameter interval $\left[0, 2\pi\right]$.
11. The point $\left(6,3\pi\right)$ is given in polar coordinates. Find its representation in rectangular coordinates.
12. Find the distance between the point $\left(-2, 1, 3\right)$ and the plane $2x-3y+z=1$.
13. Find an equation in spherical coordinates for the cylinder with rectangular equation $x^{2}+y^{2}=9$.
14. Find the domain of the parameter t of the vector function $\overline{\gamma}(t)=\left\langle\frac{1}{t}, \sqrt{t-1}, \ln t \right\rangle$
15. Give the expression for normal scalar component of acceleration a_{N} on the curve $\overline{\gamma}(t)$.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Find the derivative of $y= \frac{x^2\sqrt{2x-4}}{(x+1)^2}$ using logarithmic differentiation.

17. Find the derivative of $y=2x \coth^{-1}2x-\ln\sqrt{1-4x^2}$.

18. Use limit comparison test to determine whether the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^{5}-1}}$ is convergent or not.

19. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ absolutely convergent or not.

20. Find $\dfrac{d^{2}y}{dx^{2}}$, if $x=t^{2}-4$ and $y=t^{3}-3t$.

21. Find all the points of intersection of the curves $r=2$ and $r=4\cos2\theta$.

22. Find parametric equations for the tangent line to the helix with parametric equations $x=3\cos t, y=2\sin t, z=t$ at $t=\frac{\pi}{6}$

23. An object move with a constant speed. Show that the velocity and acceleration vectors associated with this motion are orthogonal.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 2 marks.

24. Evaluate ${\displaystyle\int \frac{1}{x\left(9+(\ln x)^2 \right)} }dx$.

25. (a) Find an approximation of the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n}}{(2n)!}$ accurate to two decimal places.

    (b) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n} n}{3n-1}$ converges or diverges.

26. Evaluate $\displaystyle\int e^{-x^2}dx.$

27. Find the arc length function $S(t)$ for the circle C in the plane described by $\overline{\gamma}(t)=2\cos t\; \bar{i}+2\sin t\;\bar{j}, \quad0\leq t\leq 2\pi$. Then find a parametrization of C in terms of S.

(2 × 10 = 20 Marks)

****