**19U503S** (Pages: 2) Name: …………………..

Reg. No: ……………….....

**FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021**

(CUCBCSS - UG)

**CC15U MAT5 B07/CC18U MAT5 B07 - BASIC MATHEMATICAL ANALYSIS**

(Mathematics - Core Course)

(2015 to 2018 Admissions – Supplementary/Improvement)

Time: Three Hours Maximum:120 marks

## SECTION A

Answer ***all*** questions. Each question carries 1 mark.

1. State the well-ordering property of Natural numbers.
2. Give an example of a denumerable set.
3. Write the Trichotomy Property of $\mathbb{R}$.
4. Find all $x$ that satisfy the inequality $|2x+5| \leq 15$.
5. If $S = \left\{\frac{1}{n} : n \epsilon\ \mathbb{N}\right\}$, then $sup\ S$ = ...............
6. Write any periodic decimal with period 2.
7. State the density theorem.
8. Give an example of a bounded sequence which is not convergent.
9. Write any monotone sequence.
10. Give an example of a closed set.
11. If $\arg z_1 = \theta_1$and $\arg z_2 = \theta_2$, then $\arg \frac{z_1}{z_2}$ = ...............
12. $\lim\limits_{z \to 2} x^2 + iy^3$ = ...............

**(12 × 1 = 12 Marks)**

## SECTION B

Answer any ***ten*** questions. Each question carries 4 marks.

13. Prove that if $a \neq 0$ and $b$ in $\mathbb{R}$ are such that $a.b = 1$, then $b = \frac{1}{a}$.
14. Prove that $|a+b| \leq\ |a|$+ $|b|$.
15. Prove that $\sqrt{2}$ is irrational.
16. State and prove Bernoulli's inequality.
17. Find the rational number represented by the decimal $1.25137137$ ... $137$ ...
18. Prove that every convergent sequence is bounded.
19. State and prove Bolzano Weierstrass theorem.
20. Evaluate $\lim \left(\frac{\sin n}{n}\right)$.

21. Prove that the intersection of finite collection of open sets in $\mathbb{R}$ is open.
22. Define cantor set.
23. Find the principal argument of $(\sqrt{3} - i)^6$
24. Prove that $Re(iz) = -Im(z)$.
25. Solve the equation $\left|e^{i\theta} - 1\right| = 2$ for $0 \le \theta \le 2\pi$.
26. Find the real and imaginary parts of $f(z) = 2z^2$-3z in Cartesian co-ordinates.

**(10 × 4 = 40 Marks)**

## SECTION C

Answer any **six** questions. Each question carries 7 marks.

27. Using mathematical induction, prove that $(n+1)! \ge 2^n, \forall\, n \in \mathbb{N}$.
28. Let $A \subseteq \mathbb{R}$. Prove that there is no surjection from $A$ to the set $\mathcal{P}(A)$ of all subsets of $A$.
29. Let S be a nonempty bounded set in $\mathbb{R}$ and for any $a \in \mathbb{R}$, $aS = \{as : s \in S\}$. Prove that $\sup(aS) = a\,Sup\,S$, if $a > 0$ and $Sup\,(aS) = a \inf S$, if $a < 0$.
30. Prove that the set of real numbers is uncountable.
31. State and prove monotone subsequence theorem.
32. State and prove Squeeze theorem.
33. Show that a subset of $\mathbb{R}$ is closed if and only if it contains all of its limit points.
34. Prove that $\sqrt{2}|z| \ge |Re\,(z)| + |Im\,(z)|$.
35. Find the three cube roots of $-8i$.

**(6 × 7 = 42 Marks)**

## SECTION D

Answer any ***two*** questions. Each question carries 13 marks.

36. State and prove nested interval property of $\mathbb{R}$..
37. (a) State and prove monotone convergence theorem

    (b) Let $Y = (y_n)$ be defined inductively by $y_1 = 1, y_{n+1} = \frac{1}{4}(2y_n + 3)$ , $\forall n \in \mathbb{N}$

    Show that $Y$is convergent and find the limit.
38. Prove that $F$ be a closed subset of $\mathbb{R}$ if and only if every convergent sequence in $F$ converges to $F$.

**(2 × 13 = 26 Marks)**

*******