19U503 (Pages: 2) Name: …………………………

Reg. No: ………………….

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS - UG)

CC19U MTS5 B07 - NUMERICAL ANALYSIS

(Mathematics - Core Course)

(2019 Admission - Regular)

Time: 2.00 Hours Maximum: 60 Marks

Credit: 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Determine the fixed points of the function $g(x) = x^2 - 2$
2. What are the drawbacks of Newton-Raphson method?
3. Write a short note on Secant Method.
4. Using the numbers $x_0 = 2, x_1 = 2.75$ and $x_2 = 4$, find the second Lagrange interpolating polynomial for $f(x) = \frac{1}{x}$
5. Using the data given in the table construct a divided difference table

x	8.1	8.3	8.6	8.7
$f(x)$	16.94	17.56	18.51	18.82

6. Using the forward-difference formula approximate the derivative of $f(x) = x^2 - \text{In } x - 1$ at $x_0 = 1$ by considering $h = 0.2$
7. Given$f(x) = e^{2x}$. By taking $h = 0.2$ and using midpoint formula find $f''(2.0)$ correct to four decimal places.
8. Approximate $\int_{e}^{e+1} \frac{1}{x\, In\, x}\, dx$ using Simpson's rule.
9. State the fundamental existence and uniqueness theorem for first order ordinary differential equations.
10. Show that the initial value problem $y' = y\, cost, 0 \le t \le 1, y(0) = 1$ has a unique solution.
11. Use Euler's method to approximate the solution $y(3)$ of the initial value problem $y' = 1 + (t - y)^2, 2 \le t \le 3, y(2) = 1$ with $h = 1$
12. Use midpoint method to approximate $y(2)$ given $y' = 1 + \frac{y}{t}, y(1) = 2$.

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** questions. Each question carries 5 marks.

13. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in$[1, 2]$. Use the Bisection method to determine an approximation to the root that is accurate to at least within 10^{-2}.
14. Use method of false position to find solution of $x^3 - 2x^2 - 5 = 0$ for accurate to within 10^{-3}.
15. Using Newton's backward-divided-difference formula evaluate P_4 (2.0) corresponding to the data given in the table

x	1.0	1.3	1.6	1.9	2.2
$f(x)$	0.7652	06201	0.4554	0.2818	0.1104

16. Values for $f(x) = xe^x$ are given in the following table. Use all the applicable three-point formulas to approximate $f'(2.0)$.

x	1.8	1.9	2.	2.1	2.2
$f(x)$	10.8893	12.7032	14.7781	17.149	19.855

Determine the actual error occurred in each case.

17. Approximate $\int_0^{\pi/4} x \sin x \; dx$ using Trapezoidal rule. Find a bound for the error using error formula and compare this to the actual error.
18. Use Taylor's method of orders two and four to approximate the solution of the initial value problem $y' = (y/t) - (y/t)^2, 1 \leq t \leq 1.2, y(1) = 1$, with $h = 0.1$
19. Use modified Euler's method to approximate the solution of the initial value problem $y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5$ with $h = 1$.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

20. Compare the results of the closed and the open Newton - Cotes formulae to approximate $\int_0^{\pi/2} (\sin x)^2 \, dx$.
21. Use Runge-Kutta method of order four to approximate the solution of the initial value problem $y' = 1 + (y/t) + (y/t)^2, 1 \leq t \leq 3, y(1) = 0,$ with $h = 1$. Compare the results to the actual values given the actual solution is $y = t \tan(\text{In } t)$.

(1 × 10 = 10 Marks)

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