21U108 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS - UG)

CC19U MTS1 B01 - BASIC LOGIC AND NUMBER THEORY

(Mathematics - Core Course)

(2019 Admission - Supplementary/Improvement)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Which of the following are propositions?
   a) Toronto is the capital of canada. b) Come in.
2. Define exclusive disjunction.
3. Write idempotent laws and identity laws of logic.
4. Rewrite the sentence "Some chalkboards are black", symbolically.
5. Write
   a) Hypothetical syllogism. b) Conjunction law.
6. Compute the first four terms of the sequence defined recursively : $a_0 = 1, a_n = a_{n-1} + n$
7. Find the quotient q and remainder r when -325 is divided by 13.
8. State the prime number theorem.
9. Find the five consecutive composite numbers less than 100.
10. Check whether the integers 8, 15 and 49 are pairwise relatively prime.
11. State Dirichlet's Theorem.
12. Give an example for diophantine equation.
13. Compute $\varphi(666)$.
14. Write the general form of linear congruence and define solution of linear congruence.
15. Find $3^{-1}(mod 4)$.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Let S be the subset of $\mathbb{N}$ that preserves the two properties:
    (i) The number $1 \in S$. (ii) For every $k \in \mathbb{N}$, if $k \in S$, then $k+1 \in S$.
    Then prove that $S = \mathbb{N}$.

17. Using the Euclidean Algorithm, Express (4076, 2076) as a linear combination of 4076 and 2076.

18. Using canonical decompositions, find the gcd of each pair; 48, 162.

19. Using recursion, evaluate the lcm of 24,28,36,40?

20. If p is a prime, then show that $(p-1)! \equiv -1 (mod p)$.

21. Compute the remainder when 43^{5555} is divided by 31.

22. Let p be a prime and a any integer such that p does not divide a. Then show that the solution of linear congruence $ax \equiv b(mod p)$ is given by $x \equiv a^{p-2} b(mod p)$.

23. Using Euler's theorem find the remainder when 245^{1040} is divided by 18.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. 1) Explain
    a) Proof of contrapositive.
    b) Direct proof.
    c) Proof by cases.
    d) Constructive existence proof.
    e) Counter example method.

    2) Prove that $\sqrt{5}$ is an irrational number.

25. State and prove Fundamental Theorem of Arithmetic.

26. a) Find the last digit in the decimal value of $1997^{1998^{1999}}$.
    b) Find the remainder when $(n^2 + n + 41)^2$ is divided by 12.

27. a) Prove that a positive integer p is a prime if and only if $\phi(p) = p - 1$.
    b) State and prove Euler's theorem.

(2 × 10 = 20 Marks)

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