21P101 (Pages: 2) Name: ……………….………

Reg. No: ……………..…...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C01 – ALGEBRA - I

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Verify whether $\phi(x, y) = (x + y, 0)$ is an isometry of the plane.
2. Find all abelian groups of order 120 up to isomorphism.
3. Compute the factor group $Z_2 \times Z_4 /\langle(0,1)\rangle$.
4. Find a maximal normal subgroup of Z. Justify your answer.
5. Find a composition series for $Z_4 \times Z_9$.
6. Find all zeroes of the polynomial x^3+2x+2 in Z_7.
7. Find the kernel of the evaluation homomorphism $\phi_i: Q[x] \longrightarrow C$.
8. Find all ideals of Z_{12}.

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT 1

9. a) Define simple group and give an example.

   b) Show that if M is a maximal normal subgroup of G, then G/M is simple.
10. Explain the falsity of converse of Lagrange’s theorem.
11. State and prove Burnsides formula.

UNIT 2

12. Show that if N is a normal subgroup of G and if H is any subgroup of G, then HN = NH.
13. Find the ascending central series of D_4.
14. Show that for a prime number p, every group of order p^2 is abelian.

UNIT 3

15. State and prove factor theorem.

16. Show that $x^3 + 3x^2 - 8$ is irreducible over Q.

17. Show that the ring Z/nZ is isomorphic to Z_n.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. a) Show that the group Z_m X Z_n is cyclic if and only if m and n are relatively prime.

    b) Show that if m divides the order of a finite abelian group G, then G has a subgroup of order n.

19. a) State and prove fundamental homomorphism theorem.

    b) Show that A_n is a normal subgroup S_n and compute S_n/A_n.

    c) Find the center of the group S_3.

20. a) State and prove first Sylow theorem.

    b) Show that no group of order 36 is simple.

21. a) State and prove division algorithm for F[x].

    b) Show that $25x^5 - 9x^4 - 3x^2 - 12$ is irreducible over Q.

    c) Find all generators of the cyclic multiplicative group of units of the field Z_5.

(2 × 5 = 10 Weightage)

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