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## SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(CBCSS - UG)
(Regular/Supplementary/Improvement)

## CC19U STA2 C02 - PROBABILITY THEORY

(Statistics - Complementary Course)
(2019 Admission onwards)
Time : 2.00 Hours

Maximum : 60 Marks
Credit: 3

Part A (Short answer questions)
Answer all questions. Each question carries 2 marks.

1. Define simple event.
2. State the a priori definition of probability.
3. What are the limitations of classical definition of probability?
4. State the properties of probability density function.
5. If the cumulative distribution function of $\backslash(\mathrm{X} \backslash)$ is $\backslash(\mathrm{F}(\mathrm{x}) \backslash)$, find the cumulative distribution function of $\backslash$ ( $\mathrm{Y}=\mathrm{X}-\mathrm{b} \backslash$ )
6. Show that $\backslash(E(a X+b)=a E(X)+b \backslash)$.
7. List any two properties of variance.
8. Define characteristic function.
9. Define Skewness.
10. What is joint probability density function?
11. Define marginal distributions.
12. If $X$ and $Y$ are independent r.v.s, show that $\operatorname{Cov}(x, y)=0$.
(Ceiling: 20 Marks)
Part B (Short essay questions - Paragraph)
Answer all questions. Each question carries 5 marks.
13. Given $\backslash(\mathrm{P}(\mathrm{A})=0.30, \mathrm{P}(\mathrm{B})=0.78 \backslash)$ and $\backslash(\mathrm{P}(\mathrm{A} \backslash$ cap B$)=0.16 \backslash)$. Find $\backslash(\backslash \operatorname{mbox}\{(\mathrm{i})\} \sim \mathrm{P}(\mathrm{A} \backslash$ cup B $) \sim \backslash \operatorname{mbox}\{(\mathrm{ii})\} \sim \mathrm{P}\left(\mathrm{A}^{\wedge} \mathrm{c} \backslash\right.$ cap B$\left.) \sim \backslash \operatorname{mbox}\{(\mathrm{iii})\} \sim \mathrm{P}(\mathrm{A} \backslash \text { cup } \mathrm{B})^{\wedge} \mathrm{c} \backslash\right)$.
14. The diameter of an electric cable, say $\backslash(\mathrm{X}, \mathrm{l})$ is assumed to be a continuous random variable with pdf, $\backslash$ ( $\mathrm{f}(\mathrm{x})=6 \mathrm{x}(1-\mathrm{x}), \sim \sim 0 \backslash$ leq $\mathrm{x} \backslash$ leq1. $) \quad$ Compute $\quad \backslash(\mathrm{P} \backslash \operatorname{left}(\quad \mathrm{X} \quad \backslash$ leqldisplaystyle\frac $\{1\}$ $\{2\} \backslash$ mid $\backslash$ displaystyle $\backslash f r a c ~\{1\}\{3\} \backslash$ leq $X \backslash$ leq $\backslash$ displaystyle $\backslash$ frac $\{2\}\{3\} \backslash$ right $).$\)
15. State and prove the multiplication theorem of probability.
16. Prove or disprove: Pairwise independence does not imply Mutual independence.
17. Define a stochastic variable and give an example.
18. Find the mgf of $\backslash(X \backslash)$ with $\operatorname{pdf} \backslash\left(f(x)=\backslash \operatorname{frac}\{1\}\{2\} \mathrm{e}^{\wedge}\{-|\mathrm{x}|\},-\backslash\right.$ infty $<\mathrm{x}<\backslash$ infty $\left.\backslash\right)$.
19. Given the joint pdf of $\backslash(X \backslash)$ and $\backslash(Y \backslash)$ as $\backslash\left(\backslash\right.$ begin $\{\operatorname{split}\} f(x, y) \&=21 x^{\wedge} 2 y^{\wedge} 3,0<x<y<1 \quad \backslash \&=0$, elsewhere lend $\{$ split $\} \backslash)$. Find the marginal distribution of $\backslash(\mathrm{X} \backslash)$ and $\backslash(\mathrm{Y} \backslash)$. Also verify whether $\backslash(\mathrm{X} \backslash)$ and $\backslash$ $(\mathrm{Y} \backslash)$ are independent?
(Ceiling: 30 Marks)
Part C (Essay questions)
Answer any one question. The question carries 10 marks.
20. (i) What do you mean by change of variable technique?
(ii) The random variable $\backslash(X \backslash)$ has the p.d.f: $\backslash\left(f(x)=e^{\wedge}\{-x\}, o \backslash l e q ~ x<\right.$ infty $\left.\backslash\right)$. Find the p.d.f of the random variable

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\backslash(\mathrm{Y}=3 \mathrm{X}+5 . \backslash)
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21. Let the joint pdf of $\backslash((\mathrm{X}, \mathrm{Y}) \backslash)$ be:
$\backslash(f(x, y)=\backslash \operatorname{left} \backslash\{\quad \backslash b e g i n\{a r r a y\}\{11\} \quad 3 x y, \& 0 \backslash$ leq $x \backslash l e q 1 ; 0 \backslash$ leq $y \backslash l e q 1 ; 0 \backslash$ leq $x+y \backslash l e q 1 \backslash$
$0, \& \backslash h b o x\{$ elsewhere. $\} \quad$ lend \{array $\} \quad$ lright. $\ \backslash \backslash$ )
Find (i) $\backslash(\mathrm{E}(\mathrm{Y} \mid \mathrm{X}=\mathrm{x}) \backslash)$; and (ii) Cor $\backslash((\mathrm{X}, \mathrm{Y}) \backslash)$.

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