19U603 (Pages: 2) Name: …………..……….

Reg. No……….……….

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(CBCSS-UG)

CC19U MTS6 B12 - CALCULUS OF MULTI VARIABLE

(Mathematics - Core Course)

(2019 Admission - Regular)

Time: 2 ½ Hours Maximum: 80 Marks

Credit: 4

Section A

Answer ***all*** questions. Each question carries 2 marks.

1. What is a function of two variables? Give an example of one by stating its rule, domain and range.
2. Find $\lim_{(x,y)\to(1,2)} \frac{2x^2-3y^3+4}{3-xy}$.
3. Let $f(x,y) = x^2 + 2y^2$. Find $f_x(2,1)$ and $f_y(2,1)$.
4. Find the differential of the function $z = 3x^2y^3 + 5xy$
5. Let $w = 2x^2y$, where $x = u^2 + v^2$ and $y = u^2 - v^2$. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.
6. Find the gradient of $f(x,y) = 2x + 3xy - 3y + 4$ at the point P(2,1).
7. Find the critical points of $f(x,y) = x^2 + y^2 - 4x - 6y + 17$.
8. Find directional derivative of $f(x,y) = 4 - 2x^2 - y^2$ at the point (1,1) in the direction of the unit vector **u** that makes an angle of $\frac{\pi}{3}$ with the positive x axis.
9. Find the limits of integration to evaluate $\iint_R \; 1 - 2xy^2 \, dA$ where $R = \{(x,y)|0 \le x \le 2, -1 \le y \le 1\}$
10. Explain why it is sometime advantageous to reverse the order of integration of an iterated integral
11. Find the equation of the cone $z = \sqrt{x^2 + y^2}$ in spherical coordinates.
12. Find Jacobian of the transformation from x-y plane to u-v plane given by $u = x - y$ and $v = 2x + y$.
13. State Divergence Theorem.
14. State Greens Theorem.
15. State Stokes Theorem.

(Ceiling: 25 Marks)

Section B

Answer ***all*** questions. Each question carries 5 marks.

16. Sketch a contour map for the surface described by $f(x,y) = x^2 + y^2$ using the level curves corresponding to $k = 0, 1, 4, 9$ & 16.
17. Let $f(x,y,z) = xe^{yz}$. Compute f_{xzy} and f_{yxz}.

18. Find equations of the tangent plane and normal line to the ellipsoid with equation $4x^2 + y^2 + 4z^2 = 16$ at the point $(1,2,\sqrt{2})$.

19. Let $w = f(x, y)$ where f has continuous second order partial derivatives.

    Let $x = r^2 + s^2$ and $y = 2rs$. Find $\frac{\partial^2 w}{\partial r^2}$.

20. Find the area of the part of the surface with equation $z = 2x + y^2$ that lies directly above the triangular region R in the plane with vertices $(0, 0), (1, 1)$ and $(0, 1)$.

21. Evaluate $\iiint_T \sqrt{x^2 + z^2}\, dV$, where T is the region bounded by the cylinder $x^2 + z^2 = 1$ and the planes $y + z = 2$ and $y = 0$.

22. Evaluate $\int_C 2x \; ds$ where C is union of the arc C_1 of the parabola $y = x^2$ from (0,0) to (1,1) followed by the line segment C_2 from (1,1) to (0,0).

23. Evaluate $\int_C \boldsymbol{F}.dr$, where $\boldsymbol{F}(x, y, z) = \cos z\, \hat{\boldsymbol{\imath}} + x^2 \hat{\boldsymbol{\jmath}} + 2y\hat{\boldsymbol{k}}$ and C is the curve of intersection of the plane $x + z = 2$ and the cylinder $x^2 + y^2 = 1$

(Ceiling: 35 Marks)

Section C

Answer any ***two*** questions. Each question carries 10 marks.

24. (a) Show that $w = 5\cos(3x + 3ct) + e^{x+ct}$ where c is a constant statisfies the wave equation $\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$.

    (b) If $\sin z = \frac{x+y}{\sqrt{x}+\sqrt{y}}$, Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. Also show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}\tan z$

25. Find the dimension of the open rectangular box of maximum volume that can be constructed from a rectangular piece of cardboard box having an area of $485\ ft^2$. What is the volume of the box?

26. Let T be the solid that is bounded by the parabolic cylinder $y = x^2$ and the plane $z = 0$ and $y + z = 1$. Find the center of mass of T, given that it has uniform density $\rho(x, y, z) = 1$.

27. Let **T** be a region bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $z = 0, x = 0$ and $x + z = 2$ and let S be the surface of T.

    If $\boldsymbol{F}(x, y, z) = xy^2\, \hat{\boldsymbol{\imath}} + \left(\frac{1}{3}y^3 - \cos xz\right) \hat{\boldsymbol{\jmath}} + xe^y\, \hat{\boldsymbol{k}}$, find $\iint_S \boldsymbol{F}.\boldsymbol{n}\; dS$.

(2 × 10 = 20 Marks)

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