FOURTH SEMESTER B.Voc. DEGREE EXAMINATION, APRIL 2022 (Regular/Supplementary/Improvement)

## CC18U GEC4 ST11 - STATISTICAL INFERENCES AND APPLICATIONS

(B.Voc. Information Technology)
(2018 Admission onwards)
Time: Three Hours

## PART A

Answer all questions. Each question carries 1 mark.
. The probability distribution of a statistic is
. If $t$ follows t -distribution with ' n ' degrees of freedom, then $Z=t^{2}$ follows $\qquad$
. A single numerical value used as an estimate of a population parameter is known as ------
4. A property of a point estimator that occurs whenever larger sample sizes tend to provide point estimates closer to the population parameter is known as $\qquad$ ---
. The method of moments was invented by $\qquad$ ----
6. $\qquad$ are the values that mark the boundaries of the confidence interval.
7. Level of significance lies between $\qquad$
8. The Neyman-Pearson lemma is used to find $\qquad$ for testing simple $\mathrm{H}_{\mathrm{o}}$ agains simple $\mathrm{H}_{1}$.
9. To test $\mathrm{H}_{0}: \mu=\mu_{0}$, when population standard deviation is unknown and the sample size is large is ------------- test
10. The hypothesis that the normal population variance has a specified value can be tested by ------------- test

## PART B

Answer any eight questions. Each question carries 2 marks.
11. Find the mean of a random variable following chi-square distribution.
12. Define $t$-distribution. Sate any two uses of $t$-distribution.
13. Define unbiasedness of estimators.
14. Write a note on the method of moments in point estimation
15. What are the properties of MLE?
for the proportion of heads.
17. Define Type I and Type II error
18. Distinguish between simple and composite hypothesis.
19. State Neymann-Pearson lemma.
20. Find the power of the test for testing $\mathrm{H}_{0}: \theta=2$ against $\mathrm{H}_{1}: \theta=3$ using a random sample from $\mathrm{U}(0, \theta)$ with a critical region $\mathrm{x}>1$.
21. What are the conditions under which a small sample $t$ test is performed to test the mean of a normal population?
22. Write the test statistic involved in small sample test of equality of variance of two normal populations.
( $8 \times 2=16$ Marks )

## PART C

Answer any six questions. Each question carries 4 marks.
23. Derive the sampling distribution of the mean of samples from a normal distribution.
24. Derive the m.g.f. of chi-square distribution.
25. State and prove sufficient conditions for a consistent estimator.
26. Define sufficient estimator. For a Poisson distribution with parameter $\lambda$, show that the sample mean is the sufficient estimator of $\lambda$.
27. Given $f(x ; \theta)=\theta(1-\theta)^{x-1} ; x=1,2,3, \ldots$ Find the moment estimator of $\theta$. Also find the moment estimate using a data given by $3,4,8,6,5,4,3,7,5$.
28. Obtain the MLE of $a$ and $b$ using the random sample $x_{1}, x_{2}, \ldots, x_{n}$ taken from a rectangular population over the interval ( $a-b, a+b$ ).
29. What are the steps involved in testing of hypothesis?
30. The heights of 10 males are found to be $70,66,59,68,62,63,61,60,59,58$ inches. Is it reasonable to think that the average height is more than 62 inches? Test at $5 \%$ level of significance.
31. A machine produced 20 defective parts in a batch of 400 . After overhauling, it produced 10 defectives in a batch of 300 . Has the machine improved at $5 \%$ level?
( $6 \times 4=24$ Marks $)$

## PART D

Answer any two questions. Each question carries 15 marks.
32. (a) Derive the statistic following:
(i) chi-square distribution
(ii) $t$ distribution (iii) F distribution. Also establish the inter relation between these sampling distribution.
(b) For a normal population, show that the sample standard deviation is a biased estimator of population standard deviation. Obtain an unbiased estimator for population variance.
33. (a) A random sample 10 observations from a normal population is given below: $76.17,65.44,94.20,72.80,75.82,92.36,73.43,72.18,84.35$ and 64.23 . Estimate the mean and standard deviation and give a $95 \%$ confidence interval for the population means.
(b) Explain various types of errors in testing of hypothesis. Let p be the probability that a coin will fall head in a single toss in order to test $H_{0}: p=0.5$ against $H_{1}: p=0.7$. The coin is tossed 5 times and $\mathrm{H}_{0}$ is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test.
34. (a) An insurance agent has claimed that the average age of policy holders who insure through him is less than the average for all agents, which is 30.5 years. A random sample of policy holders who had insured through him gave the following age distribution:

| Age | $16-20$ | $21-25$ | $26-30$ | $31-35$ | $36-40$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of persons | 12 | 22 | 21 | 24 | 1 |

(b) Explain Chi-square test of goodness of fit. Four different types ice creams A, B, C and D are assumed as preferred among adolescents in the ration 6:4:3:2. Out of a random selection of 1500 adolescents 620, 420, 290 and 170 were preferred respectively A, B, C and D. Does the observation support the assumption?
35. (a) Explain small sample test for the mean of a normal population when $\sigma^{2}$ is known and $\sigma^{2}$ is unknown.
(b) Two random samples from two normal populations are given below. Test the variance of the populations differ significantly with a significant level 0.05 .

| Sample I | 20 | 16 | 26 | 27 | 23 | 22 | 18 | 24 | 25 | 19 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sample II | 17 | 23 | 32 | 25 | 22 | 24 | 28 | 16 | 31 | 33 | 20 | 27 |

