FIFTH SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC19U MTS5 D01 / CC20U MTS5 D01 - APPLIED CALCULUS

(Mathematics – Open Course)

(2019 Admission onwards)

Time: 2 Hours   Maximum: 60 Marks

Credit: 3

Section A

Answer *all* questions. Each question carries 2 marks.

1. Find $f(x-1)$ if $f(x) = 3x^2 + \frac{1}{x} + 5$

2. Find the equation of the line passes through $(5, 1)$ with slope -1

3. Find $\lim\limits_{x \to 1} \frac{3x^3 - 8}{x - 2}$

4. If the position of an object moving along a straight line is given by $s(t) = t^3 - 3t^2 + 4t$ at time t, find its velocity and acceleration.

5. Find $\frac{dy}{dx}$ if $x^2 y + y^2 = x^3$

6. Find the critical point of the function $f(x) = 2x^3 + 3x^2 - 12x - 7$

7. Find the inflection point of the function $f(x) = x^{\frac{1}{3}}$

8. If $\log_x 27 = 3$ then $x = \cdots$

9. Find a real number x such that $2^{3-x} = 4^x$

10. Verify that $F(x) = \frac{1}{3}x^3 + 5x + 2$ is an antiderivative of $f(x) = x^2 + 5$

11. State the Fundamental theorem of Calculus

12. Let $\int_{-2}^{5} f(x)\, dx = 3$ and $\int_{3}^{5} f(x)\, dx = 7$ then $\int_{-2}^{3} f(x)\, dx = \cdots$

(Ceiling:20 Marks)

Section B

Answer *all* questions. Each question carries 5 marks.

13. A cylindrical can is to have a capacity (volume) of 24π cubic inches. The cost of the material used for the top and bottom of the can is 3 cents per square inch, and the cost of the material used for the curved side is 2 cents per square inch. Express the cost of constructing the can as a function of its radius.

14. For what value of the constant A is the following function continuous for all real x ?

$$f(x) = \begin{cases} Ax + 5 & \text{if } x < 1 \\ x^2 - 3x + 4 & \text{if } x \geq 1 \end{cases}$$

15. Evaluate

    a) $\lim\limits_{x \to 1} \frac{\sqrt{x}-1}{x-1}$

    b) $\lim\limits_{x \to +\infty} \frac{2x^2+3x+1}{3x^2-5x+2}$

16. Find the interval of increase and decrease for the function $f(x) = \frac{x^2}{x-2}$

17. The highway department is planning to build a picnic park for motorists along a major highway. The park is to be rectangular with an area of 5,000 square yards and is to be fenced off on the three sides not adjacent to the highway. What is the least amount of fencing required for this job? How long and wide should the park be for the fencing to be minimized?

18. Evaluate $\int \frac{3x+6}{\sqrt{2x^2+8x+3}} dx$

19. Suppose that when it is t years old, a particular industrial machine is generating revenue at the rate $R'(t) = 5000 - 20t^2$ dollars per year and that operating and servicing cost related to the machine are accumulating at the rate $C'(t) = 2000 + 10t^2$ dollars per year.

    a) What is the useful life of this machine?

    b) Compute the net profit generated by the machine over its period of useful life.

(Ceiling:30 Marks)

Section C
Answer any *one* question. Each question carries 10 marks.

20. Consider the function $y = \frac{u}{u+1}$, where $u = 3x^2 - 1$.

    a. Use the chain rule to find $\frac{dy}{dx}$.

    b. Find an equation for the tangent line to the graph of $y(x)$ at the point where $x = 1$.

21. A manufacturer estimates that when x units of a particular commodity are produced, the total cost will be $C(x) = \frac{1}{8}x^2 + 3x + 98$ dollars, and furthermore, that all x units will be sold when the price is $p(x) = \frac{1}{3}(75 - x)$ dollars per unit.

    a) Find the marginal cost and the marginal revenue.

    b) Use marginal cost to estimate the cost of producing the ninth unit.

    c) What is the actual cost of producing the ninth unit?

    d) Use marginal revenue to estimate the revenue derived from the sale of the ninth unit.

    e) What is the actual revenue derived from the sale of the ninth unit?

(1 × 10 = 10 Marks)

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