22P154 (Pages: 2) Name: .....

Reg.No: .....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS - PG)

CC22P MST1 C01 - ANALYTICAL TOOLS FOR STATISTICS – I

(Statistics)

(2022 Admission - Regular)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Discuss maxima and minima for $f(x,y) = 2x^4 + y^4 - 2x^2 - 2y^2$.
2. Examine the function $x^2 + y^2 + x + y + xy$ for maximum and minimum.
3. Show that $U(x,y) = \frac{1}{2}log(x^2 + y^2)$ is harmonic
4. If $f(z) = u + iv$ is analytic, then prove that both u and v are harmonic functions.
5. Show that the function $e^{1/z}$ has an isolated singularity at $z = 0$.
6. Find the Laplace transform of
   a) $sin\sqrt{t}$
   b) $e^{3x}cos2x$.
7. Define Fourier transform of a function and state its properties.

(4×2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Prove that the function $f(x,y) = 2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at the origin.
9. What do you mean by directional derivatives and total derivatives of a function? Explain.
10. Establish Taylor's theorem for a multivariable function.
11. Evaluate
    i) $\int_0^\infty e^{-3t}dt$
    ii) $\int_0^{\frac{\pi}{6}} e^{i2t}dt$.
12. Show that $\int_0^{2\pi} \frac{e^{cos\phi}cos(sin\phi)d\phi}{5-4cos(\theta-\phi)} = \frac{2\pi}{3}e^{cos\theta}cos(sin\theta)$.
13. Show that $\int_0^\infty \frac{cos\lambda x}{\lambda^2+1}d\lambda = \frac{\pi}{2}e^{-x}, x \geq 0$.

14. Find the inverse Laplace transform of $\frac{2s+1}{3s^2+5}$ and $\frac{1}{s(1+2s)}$

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Establish Taylor's theorem.

16. Find $\int_c \frac{5z-2}{z(z-1)}dz, c : |z| = 2.$

17. State Cauchy Residue theorem and prove the following
(i) $\int_0^\infty \frac{1}{1+x^2}dx = \frac{\pi}{2}$ (ii) $\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)}dx = \frac{\pi}{6}$

18. Obtain the corresponding Fourier series of the function $f(x) = \begin{cases} -k & , -\pi < x < 0 \\ k & 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$

(2 × 5 = 10 Weightage)

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