21P304 (Pages: 2) Name: ……………………..

Reg. No: …………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH3 C14 - PDE AND INTEGRAL EQUATIONS

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Differentiate between Semi Linear PDE and Quasi Linear PDE with example
2. Show that $ux + 3u\,y = 0$ has a solution of the form $u(x, y) = f\,(ax + by)$ for a proper choice of constants a, b. Find the constants.
3. Consider the equation $ux + uy = 1$, with the initial condition $u(x, 0) = f\,(x)$. What are the projections of the characteristic curves on the (x, y) plane?
4. Show that the equation $uxx + 6uxy - 16u\,yy = 0$ is hyperbolic
5. Let u be a function in $C^2(D)$ satisfying the mean value property at every point in D. Then prove that u is harmonic in D
6. Define Poisson equation and classify problems with Poisson equation based on its boundary condition.
7. Explain separable kernel with an example.
8. If $y''(x) = F(x)$ and y satisfies the initial condition $y(0) = y_0$ and $y'(0) = y_0'$ Show that $y(x) = \int_0^x (x - \xi)\,f(s)d\xi + y_0'x + y_0$.

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT-1

9. Solve the equation. $u_x + u_y = 2$ Subject to the initial condition $u(x, 0) = x^2$.
10. Prove that the type of a linear second order PDE in two variables is invariant under a change of co-ordinates.
11. Let $u(x, t)$ be a solution of the wave equation $u_{tt} - c^2u_{xx} = 0$ which is defined in the whole plane. Assume that u is a constant on the line $x = 2 + ct$. Prove that $u_t + cu_x = 0$.

UNIT-2

12. Prove the solution is unique for the following problem.

$$u_t - ku_{xx} = F(x,t) \ ,0 < x < L \,, t > 0$$
$$u(0,t) = a(t) \quad , \quad u(L,t) = b(t) \ \ , t \geq 0$$
$$u(x,0) = f(x)\,, \qquad 0 \leq x \leq L$$

13. State and prove the strong maximum principle.

14. Prove that a necessary condition for the existence of a solution to the Neumann problem is $\int_{\partial D} g(x(s), y(s)).\,ds = \int_D F(x,y)dxdy$ where$(x(s), y(s))$ is a parameterization of ∂D.

UNIT-3

15. Find the Resolvent kernel of the equation

$$y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi) y(\xi) d\xi$$

16. Transform $y'' + xy = 1\,, y(0) = 0\,, y(1) = 1$ into an integral equation using green's function.

17. Prove that the eigen values of Fredholm equation with real symmetric kernel are all real.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. (a) Compute the function $u(x,y)$ satisfying the eikonal equation ${u_x}^2 + {u_y}^2 = n^2$ and the initial condition $u(x,1) = n\sqrt{1+x^2}$ (n is a constant).

    (b) Consider the Tricomi equation $u_{xx} + u_{yy} = 0\,, x < 0$. Find a mapping $q = q(x,y), r = r(x,y)$ that transforms the equation into its canonical form and present the equation in this coordinate system.

19. Solve the heat conducting problem.

$$u_t - u_{xx} = 0\,, 0 < x < \pi\,, t > 0$$
$$u(0,t) = (\pi, t) = 0\,, t \geq 0$$
$$u(x,0) = f(x) = \begin{cases} x & 0 \leq x \leq \pi/2 \\ \pi - x & \pi/2 \leq x \leq \pi \end{cases}$$

20. State and prove Mean value theorem using poisson formula.

21. Deduce the Neumann series for a Fredholm integral equation and express the resolvent kernel as its solution.

(2 × 5 = 10 Weightage)

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