22U209S (Pages: 2) Name: ....

Reg. No: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(CUCBCSS-UG)

CC15U MAT2 C02 - MATHEMATICS

(Mathematics – Complementary Course)

(2015 to 2018 Admissions – Supplementary/Improvement)

Time: Three Hours Maximum: 80 Marks

PART A

Answer ***all*** questions. Each question carries 1 mark.

1. If $\sinh x = -3/4$, then $\cosh x = \cdots$
2. Evaluate $\int 4e^x \sinh x \, dx$
3. Evaluate $\int_1^\infty \frac{1}{x^2} dx$
4. Evaluate $\lim_{n\to\infty} \left(\frac{n-1}{n}\right)$
5. Discuss the convergence of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$
6. Show that $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ converges absolutely for all values of x.
7. Find a Maclaurin series for $f(x) = e^{x/2}$
8. Graph the set of points whose polar coordinates satisfy the equation $0 \le r \le 1$
9. Replace the Cartesian equation $y = x$ by equivalent polar equation.
10. Identify the conic $r = \frac{4}{1+\sin\theta}$
11. Convert the cylindrical coordinates $\left(2, \frac{\pi}{3}, 1\right)$ into rectangular coordinates.
12. Find the domain of the function $f(x, y, z) = xy \ln z$

(12 × 1 = 12 Marks)

PART B

Answer any ***nine*** questions. Each question carries 2 marks.

13. Differentiate $f(x) = x \sinh x - \cosh x$ w.r.t x.
14. Show that $\operatorname{sech}^{-1} x = \cosh^{-1}\left(\frac{1}{x}\right)$
15. Show that $\int_{-\infty}^{\infty} \frac{2x}{1+x^2} dx$ is divergent
16. Evaluate $\lim_{n\to\infty} \frac{n!}{n^n}$
17. Find the sum of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots$
18. Discuss the convergence of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$
19. Find the Taylor series generated by $f(x) = \sin x$ at $x = \frac{\pi}{2}$.

20. Identify the graph of $r\sin\theta = -1$

21. Show that $\left(\frac{1}{3}, \frac{3\pi}{2}\right)$ lies on the curve $r = -\sin\left(\frac{\theta}{3}\right)$

22. Find a cartesian equation for the surface $\rho = \cos\varphi$

23. Show that the function $f(x,y) = \frac{x^4-y^2}{x^4+y^2}$ has no limit as $(x,y) \to (0,0)$.

24. Find the linearization of $f(x,y) = x^2 + y^2 + 1$ at the point $(1,1)$.

(9 × 2 = 18 Marks)

PART C

Answer any ***six*** questions. Each question carries 5 marks.

25. Find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$ and the lines $y = 0, x = 2$ about the *x*-axis.

26. Find the length of $x = \frac{y^3}{3} + \frac{1}{4y}$ from $y = 1$ to $y = 3$.

27. Discuss the convergence of $\int_{-\infty}^{\infty} \frac{1}{1+x^2}dx$

28. Show that $\left\{(-1)^{n+1}\frac{n-1}{n}\right\}$ diverges.

29. Discuss the convergence of $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$

30. For what values of *x* does the series $\sum_{n=1}^{\infty} n!\,x^n$ converges?

31. Find the area of the Lemniscate of Bernouilli $r^2 = \cos 2\theta$

32. Show that the function $f(x,y) = \begin{cases} \frac{2x^2y}{x^4+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ is continuous at every point except at the origin.

33. Verify that $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$, when $w = x^y + \sin(xy)$

(6 × 5 = 30 Marks)

PART D

Answer any ***two*** questions. Each question carries 10 marks.

34. Find the area of the surface generated by revolving the curve $x = 2\sqrt{4-y},\ 0 \leq y \leq \frac{15}{4}$, about the *y*-axis.

35. Discuss the converges of

(a) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ (b) $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

36. Find the length of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$.

(2 × 10 = 20 Marks)

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