21U409S (Pages: 2) Name: ……………………..

Reg. No: …………………..

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(CUCBCSS-UG)

CC15U MAT4 C04 - MATHEMATICS – IV

(Mathematics – Complementary Course)

(2015 to 2018 Admissions – Supplementary/Improvement)

Time: Three Hours Maximum: 80 Marks

PART – A

Answer ***all*** questions. Each question carries 1 mark.

1. Check whether the functions $\cos x$ and $\sin x$ are independent or not.
2. Find the general solution of $y'' - y = 0$.
3. Write the characteristic equation of $x^2y'' + 3xy' + 7y = 0$.
4. Find the Wronskian of the solutions e^x and xe^x.
5. $\mathcal{L}(\sinh at) = \cdots$
6. $\mathcal{L}^{-1}\left(\frac{1}{s^3}\right) = \cdots$
7. Give an example for a periodic function with its fundamental period.
8. Find a_0 for representing the function $f(x) = x$ as a Fourier series in the interval $[-\pi, \pi]$.
9. Prove that the function $f(x) = x \sin x$ is an even function.
10. Write the order and degree of the partial differential equation $\left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial u}{\partial y} = 0$.
11. Write the one-dimensional wave equation.
12. Write the formula for finding k in Runge - Kutta method.

(12 × 1 = 12 Marks)

PART – B

Answer any ***nine*** questions. Each question carries 2 marks.

13. Solve $y'' - 4y' + 4y = 0$.
14. Apply the differential operator $(D + 5)^2$ to the function $y = 5x + \sin 5x$.
15. Give the form of choice of y_p for solving the differential equation $y'' + 2y' - 35y = 5e^{3x} + 2\sin 5x$.
16. Find the Laplace transform of the function $3t + \cos 2t$.
17. Find the inverse Laplace transform of $\frac{1}{(s+1)(s-1)}$.
18. State Second shifting theorem.

19. Find b_n for representing the function $f(x) = \begin{cases} -k & when -\pi < x < 0 \\ k & when\ 0 < x < \pi \end{cases}$ as a Fourier series in the interval $(-\pi, \pi)$.

20. Write the general form of Fourier cosine series and Fourier sine series.

21. Verify that $u = e^x \cos y$ is a solution of the Two-dimensional Laplace equation.

22. Solve the partial differential equation $u_y = u$.

23. Using Picard's method find $y_2(x)$, for the initial value problem $y' = x + y, y(0) = 1$.

24. Apply Euler's method to solve the initial value problem $y' = (y + x)^2, y(0) = 0$ by computing y_1 and y_2 with $h = 0.1$.

(9 × 2 = 18 Marks)

PART – C

Answer any ***six*** questions. Each question carries 5 marks.

25. Solve the initial value problem: $y'' + 4y' + 4y = 0, y(0) = 1, y'(0) = 1$.

26. Find the general solution of $x^2y'' - 4xy' + 6y = 0$.

27. Find $\mathcal{L}(t \cos \omega t)$.

28. Find the inverse Laplace transform of $\frac{6s-4}{s^2-4s+20}$.

29. Using the method of convolution, find $\mathcal{L}^{-1}\left(\frac{1}{(s^2+1)^2}\right)$.

30. Obtain the Fourier series for $f(x) = \pi x$ in $[0,2]$.

31. Find the half range sine series of the function $f(x) = \pi - x, 0 < x < \pi$.

32. Find the solution $u(x, y)$ of the partial differential equation $u_x = 2u_y + u$ by separating variables.

33. Find an upper bound for the error incurred in estimating the integral $\int_0^\pi x \sin x \; dx$ using Trapezoidal rule with $n = 10$ steps.

(6 × 5 = 30 Marks)

PART-D

Answer any ***two*** questions. Each question carries 10 marks.

34. Solve the initial value problem:

$$y'' + 2y' + 5y = 1.25e^{0.5x} + 40 \cos 4x - 55 \sin 4x, y(0) = 0.2, y'(0) = 60.1$$

35. (i) Prove that $\mathcal{L}(\sin at) = \frac{a}{s^2+a^2}$.

    (ii) Solve the integral equation $y(t) = t + \int_0^t y(u) \sin(t - u) \; du$.

36. Find the Fourier series representing $f(x) = x$ in the interval $[-\pi, \pi]$. Hence, deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$.

(2 × 10 = 20 Marks)

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