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# THIRD SEMESTER B.Voc. DEGREE EXAMINATION, NOVEMBER 2022 

(Information Technology)

## CC18U GEC3 ST08 - PROBABILITY DISTRIBUTIONS

(2018 to 2020 Admissions - Supplementary/improvement)
Time: Three Hours
Maximum: 80 Marks

Section A (One word questions)
Answer all questions. Each question carries 1 mark.
Fill up the blanks:

1. $\quad \mathrm{X}$ is a Poisson random variable with mean 2. Then $\mathrm{V}(\mathrm{X})=$ $\qquad$
2. In the expansion of $\mathrm{M}_{\mathrm{x}}(\mathrm{t})$ the coefficient of $\frac{t^{r}}{r!}$ is $\qquad$
3. If $X$ and $Y$ are two random variables with bivariate distribution function $F(x, y)$, then the value of $\mathrm{F}(\infty, \infty)$ is $\qquad$
4. Convergence in probability is also known as $\qquad$
5. If $X \sim N(12,4)$, then $E(X)$ $\qquad$
Write true or false:
6. Mean of binomial distribution is 5 and variance is 10 .
7. We can use Chebyshev's inequality to prove the weak law of Large Numbers.
8. The moment generating function is always exists.
9. The cumulative distribution function $\mathrm{F}(\mathrm{x}, \mathrm{y})$ lies between zero and one.
10. If X and Y are two independent random variables, then $\operatorname{Cov}(\mathrm{X}, \mathrm{Y})=0$.
( $10 \times 1=10$ Marks)
Section B (One Sentence questions)
Answer any eight questions. Each question carries 2 marks.
11. Define conditional expectation.
12. Define Bernoulli distribution.
13. Give any two properties of joint distribution function.
14. Define mean and variance of a random variable.
15. Explain the concept of 'convergence in probability'.
16. Define characteristic function of a random variable.
17. Explain joint probability density function.
18. Define $\mathrm{r}^{\text {th }}$ central moment of a random variable.
19. Obtain the mean of continuous uniform distribution.
20. Find the mode of a binomial distribution with parameters ( $15,0.25$ ).
21. State weak law of large numbers.
22. Define standard normal distribution.

## Section C (Short Essay questions)

Answer any six questions. Each question carries 4 marks.
23. If $M_{x}(t)$ is the MGF of a random variable $X$, explain how to obtain the moments of $X$ from $M_{x}(t)$.
24. State and prove additive property of normal distribution.
25. Define moment measures of skewness and kurtosis.
26. Define exponential distribution. Obtain its MGF.
27. Find K so that $\mathrm{f}(\mathrm{x}, \mathrm{y})=\mathrm{K}\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right), \quad 0 \leq x \leq 2, \quad 1 \leq y \leq 4$

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=0 \text {, elsewhere }
$$

will be a bivariate probability density function.
28. State and prove addition and multiplication theorems on expectation.
29. State and prove lack of memory property of geometric distribution.
30. State (i) Bernoulli law of large numbers.
(ii) Lindberg Levy form of central limit theorem. What are its assumptions?
31. Find the least value of probability $\mathrm{P}(1 \leq \mathrm{X} \leq 7)$ where X is a random variable with $\mathrm{E}(\mathrm{X})=4$ and $\mathrm{E}\left(\mathrm{X}^{2}\right)=2$.

Section D (Essay questions)
Answer any two questions. Each question carries 15 marks.
32. Given the following table:

| $x$ | -3 | 6 | 9 |
| :---: | :---: | :---: | :---: |
| $p(x)$ | $1 / 6$ | $1 / 2$ | $1 / 3$ |

Compute (i) $\mathrm{E}(\mathrm{X})$, (ii) $\mathrm{E}(2 \mathrm{X}+1)$, (iii) $\mathrm{E}(2 \mathrm{X}+1)^{2}$, (iv) $\mathrm{E}\left(\mathrm{X}^{2}\right)$, (v) $\mathrm{V}(\mathrm{X})$,(vi) $\mathrm{V}(3 \mathrm{X}+4)$, (vii) MGF
33. (i) Derive the MGF of a normal distribution with parameters $\mu$ and $\sigma^{2}$.
(ii) X is normally distributed and the mean of X is 12 and variance is 16 .

Find $\mathrm{P}(\mathrm{X} \geq 12)$.
34. State and prove Chebychev's inequality.
35. Two random variables X and Y have the following joint probability density function: $(x, y)=\mathrm{K}(4-\mathrm{x}-\mathrm{y}) ; \quad 0 \leq \mathrm{x} \leq 2,0 \leq \mathrm{y} \leq 2$, Find
i) The constant K
ii) The marginal p.d.fs of X and Y
iii) $V(X)$
iv) Are X and Y independent?

