FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS1 C01 / CC20U MTS1 C01 - MATHEMATICS - I

(Mathematics - Complementary Course)

(2020 Admission onwards)

Time : 2.00 Hours   Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer *all* questions. Each question carries 2 marks.

1. Find $\lim\limits_{x \to 3} \dfrac{8x^2}{1 + \sqrt{x}}$

2. Find $\dfrac{d}{dx}\left(4x^9 - 6x^5 + 3x\right)$

3. Find $\dfrac{d}{dx}\left(\dfrac{2x + 1}{x^2 - 2}\right)$

4. Let $f(x) = (x - 2)^3$ and $g(x) = x^3$. Find $f \circ g$ and $g \circ f$.

5. If $xy + \sqrt{x^2 - y} = 7$, compute $\dfrac{dy}{dx}$ using implicit differentiation.

6. Find $\displaystyle\int \left(\dfrac{t^3 - 8t + 1}{t^{3/2}}\right) dt$

7. Suppose that f is continuous on $[0, 3]$, that f has no roots on the intreval and that $f(0) = 1$. Prove that $f(x) > 0$ for all x in $[0, 3]$.

8. Using algebra alone, show that $f(x) = x^2$ is increasing at $x_0 = 2$.

9. Use the second derivative test to analyze the critical points of the function $f(x) = 6x^5 - x + 20$

10. State Horserace Theorem.

11. Find the sum $1 + 2 + \ldots + 25$

12. An object moving in a straight line has velocity $v = 2t^3 + t^4$ at time t. How far does the object travel between $t = 0$ and $t = 2$?

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer *all* questions. Each question carries 5 marks.

13. Use the formal definition find the derivative of $f(x) = \dfrac{10}{x}$ for $x \neq 0$.

14. Find the equation of the line tangent to the graph of $y = \sqrt{x} + \dfrac{1}{2(x+1)}$ at $x = 1$.

15. Identify the parametric curve $x = at + b, \quad y = ct + d$ where a, b, c, d are all constants. What is its slope?

16. Find the critical points, endpoints, maximum and minimum points and values of the function $f(x) = x^3 + 3x^2 - 3x + 1$ on the intreval $[-1, 2]$.

17. Find $\lim\limits_{x \to 0} \left(\dfrac{1}{x \sin x} - \dfrac{1}{x^2} \right)$

18. Compute the exact value of $\int_0^1 (x^3) dx$ by using Reimann sums and the formula
$$1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + N^3 = \left(\dfrac{(N(N+1))}{2} \right)^2 .$$

19. Find the area of the region between the graphs of x and $x^2 + 1$ on $[-2, 2]$.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any *one* question. The question carries 10 marks.

20. (a) An oil slick has area $y = 30x^2 + 100x$ square meters x minutes after a tanker explosion. Find the average rate of change in area with respect to time during the period from $x = 2$ to $x = 3$ and from $x = 2$ to $x = 2.1$. What is the instantaneous rate of change of area with respect to time at $x = 2$?

    (b) Compute the second derivative $\dfrac{d^2}{dx^2} \left(\dfrac{x^2}{x-1} \right)$.

21. The region under the graph of $cos x + 1$ on $[0, 2\pi]$ is revolved about the x axis. Using disk method find its volume.

(1 × 10 = 10 Marks)

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