23P156 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC22P MST1 C03 - DISTRIBUTION THEORY

(Statistics)

(2022 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. If X and Y are independent and identically distributed geometric random variables. Find the distribution of Max(X, Y).
2. State and prove the recurrence relation for cumulants of power series distribution.
3. (a) Define Pareto distribution and mention its important characteristics.
   (b) State any characterisation of Weibul distribution.
4. If X and Y are independent exponential random variables with parameter one. Show that $\frac{X}{X+Y}$ has $U(0,1)$ distribution.
5. Define a finite mixture of probability density function. Verify that a mixture of pdf's satisfies the properties of a pdf.
6. Define Order Statistics. Derive the distribution of n^{th} order statistic.
7. Define non central 't' distribution. Show that square of non central 't' follows non central 'F' distribution.

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Define m.g.f of a random variable. Find the mgf of i) $Y = aX + b$ ii) $Y = \frac{X-m}{\sigma}$
9. Let X be a random variable with p.m.f $P(X=j) = p_j,\ j = 0,1,2,\ldots$. Define $q_j = P(X > j)$, $j = 0,1,2,\ldots$ and $Q(s) = \sum_{j=0}^{\infty} q_j s^j$. Show that $Q(s) = \frac{1-P(s)}{1-s}$ for $|s| < 1$ where P(s) is the p.g.f of X.
10. If X and Y are independent standard normal variates then obtain the distribution of $U = \frac{X}{|Y|}$.
11. Write down the Beta probability functions of the second kind. Derive its arithmetic mean and harmonic mean.

12. If X and Y are standard normal variates with correlation coefficient ρ,

(a) Prove that X+Y and X-Y are independent.

(b) $Q = \frac{X^2-2\rho XY+Y^2}{1-\rho^2}$ is distributed as chi-square.

13. State Chebychev's inequality. If X be distributed with pdf $f(x) = 1,\ 0 < X < 1$. Prove that $P(|X - \frac{1}{2}| < 2\sqrt{\frac{1}{2}}) \geq \frac{3}{4}$.

14. If X is a Chi-square variate with n d.f then prove that for large n, $\sqrt{2X}$ follows normal $N(\sqrt{2n}, 1)$ distribution.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Define negative binomial distribution. Obtain its moment generating function.Hence obtain mean and variance.

16. Write down the differential equation which generates the Pearsonian distribution. Deduce the Gamma and Beta distributions as members of the family. Also identify a distribution which is not a member of the system.

17. a) Show that Var(X)= E(Var(X|Y)) + Var(E(X|Y)).

b) Derive the joint distribution of $X_{(r)}$ and $X_{(s)}$, the r^{th} and s^{th} order statistics.

18. In sampling from a normal population, show that the sample mean $\bar{X}$ and the sample variance S^2 are independently distributed.

(2 × 5 = 10 Weightage)

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