CRYPTOGRAPHY

A project report submitted to Christ College (Autonomous) in partial fulfilment of requirement for the award of the B.Sc. Degree programme in Mathematics

By

Aswathy P Y(CCAUSMT045)

DEPARTMENT OF MATHEMATICS

CHRIST COLLEGE (AUTONOMOUS)

IRINJALAKUDA

2023

CERTIFICATE

This is to certify that the project entitled "CRYPTOGRAPHY" submitted to the Department of Mathematics in partial fulfillment of the requirement for the award of the B.Sc. Degree programme in Mathematics, is a bonafide record of original research work done by Aswathy P Y (CCAUSMT045)during the period of their study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda under my supervision and guidance during the year 2022-2023.

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Date: 18-04-2023

DECLARATION

We hereby declare that the project work entitled "CRYPTOGRAPHY" submitted to the Christ College (Autonomous) in partial fulfillment of the requirement for the award of the B.Sc. Degree programme in Mathematics is a record of original project work done by us during the period of our study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda.

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First, there are no words to adequately acknowledge the wonderful grace that our redeemer has given us. There are many individuals who have come together to make this project a reality. We greatly appreciate the inspiration, support and guidance of all those people who have been instrumental for making this project a success.

We express our deepest thanks to our guide Dr. JOJU K T, Department of Mathematics, Christ College (Autonomous), Irinjalakuda, who guided us faithfully through this entire project. We have learned so much from him both in the subject and otherwise. Without his advice, support and guidance, it find difficult to complete this work.

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JORDAN CANONICAL FORM

Project report submitted to Christ College (Autonomous) in partial fulfilment of requirement for the award of the B.Sc. Degree Programme in Mathematics

By

Devika Linson (CCAUSMT056)

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This is to certify that the project entitled "JORDAN CANONICAL FORM" submitted to the Department of Mathematics (Unaided) in partial fulfilment of the requirement for the award of the B.Sc. Degree programme in Mathematics, is a bonafide record of original research work done by Devika Linson (CCAUSMT056) during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda under my supervision and guidance during the year 2022-2023.

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Project report submitted to Christ College (Autonomous) in partial fulfilment of requirement for the award of the B.Sc. Degree Programme in Mathematics

By

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CONTENT

INTRODUCTION

Any linear transformation can be represented by its matrix representation. In an ideal situation, all linear operators can be represented by a diagonal matrix. However, in the real world, there exist many linear operators that are not diagonalizable. This gives rise to the need for developing a system to provide a beautiful matrix representation for a linear operator that is not diagonalizable.

Matrices are "natural" mathematical objects: they appear in connection with linear equations, linear transformations, and also in conjunction with bilinear and quadratic forms, which were important in geometry, analysis, number theory, and physics. Matrices as rectangular arrays of numbers appeared around 200 BC in Chinese mathematics, but there they were merely abbreviations for systems of linear equations. Matrices become important only when they are operated on added, subtracted, and especially multiplied; more important, when it is shown what use, they are to be put to.

Cayley advanced considerably the important idea of viewing matrices as constituting a symbolic algebra. In particular, his use of a single letter to represent a matrix was a significant step in the evolution of matrix algebra. But his papers of the 1850s were little noticed outside England until the 1880s. During the intervening years deep work on matrices (in one guise or another) was done on the continent, by Cauchy, Jacobi, Jordan, Weierstrass, and others. They created what may be called the spectral theory of matrices: their classification into types such as symmetric, orthogonal, and unitary; results on the nature of the Eigen values of the various types of matrices; and, above all, the theory of canonical forms for matrices the determination, among all matrices of a certain type, of those that are canonical in some sense. An important example is the Jordan canonical form, introduced by Weierstrass (and independently by Jordan), who showed that two matrices are similar if and only if they have the same Jordan canonical form.

CHAPTER 1
BASIC CONCEPTS OF MATRICES

1.1 Matrix

A matrix is a rectangular array of numbers or functions arranged in horizontal rows and vertical columns.

Example:

$$[1 \quad 2 \quad 3]_{1\times 3} \qquad \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix}_{2\times 2}$$

1×3 and 2×2 are called the order of the matrix. 1×3 means 1 row and 3 columns. The entries of a matrix are called element.

In general, a matrix A of order $m \times n$ is given by

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

Any element having its row index equal to its column index is a diagonal element. If the matrix has as many rows as columns, $m = n$, it is called a square matrix; In general, it is written as

$$A_{n\times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & . & . & a_{2n} \\ a_{31} & a_{32} & a_{33} & . & . & a_{3n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & a_{n3} & . & . & a_{nn} \end{bmatrix}$$

In this case, the elements $a_{11}, a_{22}, a_{33}, \dots, a_{nn}$, lie on and form the main (or principal) diagonal.

1.2 Matrix Multiplication

1. The product of two matrices AB is defined if the number of columns of A equals the number of rows of B.
2. If the product AB is defined, then the resultant matrix will have the same number of rows as A and the same number of columns as B.

3. If the product $AB = C$ is defined, where C is denoted by $[c_{ij}]$, then the element c_{ij}, is obtained by multiplying the elements in i^{th} row of A by the corresponding element in the j^{th} column of B and adding.

In general matrix multiplication is not a commutative operation, that is $AB \neq BA$.

1.3 Vectors

A vector is a $1 \times n$ or $n \times 1$ matrix. A $'1 \times n'$ matrix is called a row vector while an a $'n \times 1'$ called a column vector. The elements are called the component of the vector while the number of components in the vector is n.

Magnitude or length of the row (column) vector

$$y = [y_1 \quad y_2 \cdots y_{n-1} \quad y_n]$$

is defined as follows:

$$\|y\| = \sqrt{y_1^2 + y_2^2 + \cdots + y_{n-1}^2 + y_n^2}$$

1.4 Inverse of a matrix and Diagonal matrix

An inverse of a $n \times n$ matrix A is a $n \times n$ matrix B having the property that $AB = BA = I$. B is called an inverse of A and is usually denoted by A^{-1}. If a square matrix A has an inverse, it is said to be invertible or non-singular. Inverse of a matrix are only defined for square matrices.

A diagonal matrix is a square matrix all of whose elements are zero except possibly those on the main diagonal. The inverse of a diagonal matrix exists only for the diagonal matrix having non-zero elements on its main diagonal and also the inverse matrix a diagonal matrix whose diagonal elements are the reciprocals of the corresponding diagonal elements of D. That is if,

$$D = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}$$

Its inverse matrix is,

$$D = \begin{pmatrix} \frac{1}{\lambda_1} & 0 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{\lambda_n} \end{pmatrix}$$

1.5 Eigen values and Eigen vectors

A non-zero vector X is an eigen vector (or characteristic vector) of a square matrix A, if there exists a scalar λ. such that

$$AX = \lambda X$$

Then, λ is an eigen value (or characteristic value) of A.

1.5.1 Linearly independent Eigen vectors

The vectors $U_1, U_2, \dots, U_n$ are said to be linearly independent, If the equation

$$C_1U_1 + C_2U_2 + \cdots + C_nU_n = 0 \text{ implies that } C_1 = C_2 = C_3 = \cdots = C_n = 0$$

Remark:

Eigen vectors corresponding to distinct eigen values are linearly independent.

CHAPTER -2

DIAGONALIZABLE MATRICES AND GENERALIZED EIGENVALUES

2.1 Similar Matrices

A matrix A is similar to matrix B, if there exists an invertible matrix P such that

$$A = P^{-1}BP \qquad \text{……. (1)}$$

If we pre-multiply (1) by P, it follows that A is similar to B, if and only if there exists a non – singular matrix P such that

$$PA = BP \qquad \text{…… (2)}$$

Furthermore, if we post multiply (2) by P^{-1},

We see that A is similar to B, if and only if B is similar to A.

Example:

Determine whether $A = \begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix}$ is similar to $B = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$

Solution:

A will be similar to B if and only if there exists a non-singular matrix P such that $PA = BP$ is satisfied. Designate P by $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Then, $PA = BP$ implies that

$$\begin{bmatrix} 4a - 2b & 3a - b \\ 4c - 2d & 3c - d \end{bmatrix} = \begin{bmatrix} 5a - 4c & 5b - 4d \\ 3a - 2c & 3b - 2d \end{bmatrix}$$

Equating corresponding elements, we find that the elements of P must satisfy the 4 equations:

$$-a - 2b + 4c = 0 \qquad 3a - 6b + 4d = 0$$

$$-3a + 6c - 2d = 0 \qquad -3b + 3c + d = 0$$

A solution to this set of equations is,

$$a = \frac{-2}{3}d \qquad b = \frac{1}{3}d \qquad c = 0 \quad \text{and d is arbitrary number.}$$

Thus,

$$P = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{d}{3}\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$$

P is invertible if d≠0. Thus, by choosing d≠0, we obtain an invertible matrix P that satisfy $PA = BP$, which implies that A is similar to B.

2.1.1 Theorem

Similar matrices have the same characteristic equation (and therefore, the same Eigen values).

Proof:

Let A and B be similar matrices. The characteristic equation of A is

$$|A - \lambda I| = 0$$

While the characteristic equation of B is

$$|B - \lambda I| = 0$$

Thus, it is sufficient to show that

$$|A - \lambda I| = |B - \lambda I|$$

Since A is similar to B, there must exist a non- singular matrix P such that $A = P^{-1}BP$ is satisfied, and since P is invertible, we can write

$$\lambda I = \lambda P^{-1}P = P^{-1}\lambda P$$

Then,

$$|A - \lambda I| = |P^{-1}BP - \lambda I| \;\; = |P^{-1}BP - P^{-1}\lambda IP|$$

$$= |P^{-1}(B - \lambda IP)| = |P^{-1}||(B - \lambda I)||P| = |(B - \lambda I)|$$

(Here we use the properties that $|AB| = |A||B|$ and if A is invertible $|P^{-1}| = \frac{1}{p-1}$)

Example:

Determine whether $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ is similar to $B = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

Solution:

Characteristic equation of A is $|A - \lambda I| = 0$

That is,

$$|A - \lambda I| = \begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix} = (1-\lambda)(3-\lambda) - 8 = \lambda^2 - 4\lambda - 5 = 0$$

The characteristic equation B is

$$|B-\lambda I| = \begin{bmatrix} 1-\lambda & 4 \\ 3 & 2-\lambda \end{bmatrix}$$

$$= (1-\lambda)(2-\lambda) - 12 = \lambda^2 - 3\lambda - 10 = 0$$

Since these two equations are not identical. It follows from theorem that A is not similar to B.

NOTE:

Consider the matrices $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

Although both matrices have the same characteristic equation, namely $(\lambda - 2)^2 = 0$. But they are not similar. If two matrices do not have the same characteristics then we can state, categorically, that they are not similar. However, if two matrices have the same characteristic equation, they may or may not be similar.

2.2 Diagonalizable Matrices

A matrix is said to be diagonalizable if it is similar to a diagonal matrix. If a matrix is similar to a diagonal matrix D, then the form of D are precisely the elements of the main diagonal of D, it follows that the main diagonal of D must consist of the eigen values of A.

2.2.1Two Important Properties Of Matrix Factoring

Property 1

Let $B = [b_{1,} b_2, b_3, \dots., b_n]$ be an $n \times n$ matrix, where b_j $(j = 1,2,3,\dots)$ is the jth column of B considered as a vector. Then for any $n \times n$ matrix A,

$$\mathrm{AB} = \mathrm{A}[\mathrm{b}_1 \;\; b_2 \;\; b_3 \;\; \dots \;\; b_n] = [A\mathrm{b}_1 \;\; Ab_2 \;\; Ab_3 \;\; \dots \;\; Ab_n]$$

Property 2

Designate the $n \times n$ matrix B by $[b_1 \;\; b_2 \;\; b_3 \dots \; b_n]$ as in property1 and let $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ represent scalars. Then

$$[\lambda_1 b_1 \;\; \lambda_2 b_2 \;\; \lambda_3 b_3 \;\; \dots \;\; \lambda_n b_n] = [b_1 \;\; b_2 \;\; b_3 \;\; \dots \;\; b_n] \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$$

$$= B \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$$

2.2.2 Theorem

An $n \times n$ matrix A is diagonalizable if and only if its possess n linearly independent eigen vectors. The inverse of the matrix P is a modal matrix of A.

Proof:

Let A be a $n \times n$ matrix that has 'n' linearly eigen vectors $X_1, X_2 X_3, \dots, X_n$ which corresponding to the eigen values $\lambda_{1,} \lambda_2, \lambda_3, \dots. \lambda_n$ define,

$M = [X_1\, X_2 \dots . X_n]$ and

$$D = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$$

Here M is called a modal matrix for A, and D is called a spectral matrix of A. Note that sine eigen vectors themselves are not unique, and since both the columns of M and D may be interchanged. It follows that both M and D not unique. Using property 1 and 2 and the fact that $X_j (j = 1,2,3, \dots)$ is an eigen vector of A, we have that

$$\begin{aligned} AM &= A[X_1\, X_2 \dots X_n] \\ &= [AX_1\, AX_2 \dots AX_n] \\ &= [\lambda_1 X_1\ \ \lambda_2 X_2 \dots . \lambda_n X_n] \qquad \dots\dots. (3) \\ &= [\, X_1\, X_2 \dots X_n] D \\ &= MD \end{aligned}$$

Since the column of M are linearly independent, it follows that the column rank of M is n, the determinant of M is non zero, and M^{-1} exists. Pre multiplying by M^{-1} we obtain,

$$D = M^{-1} A M \qquad \dots\dots (4)$$

which implies that D is similar to A. Furthermore, by defining $P = M^{-1}$, it follows that

$$A = P^{-1} D P = M D M^{-1} \qquad \dots\dots (5)$$

Which implies that A is similar to D. Since we can retrace our steps and show that if (5) is satisfied, then P must be M^{-1}.

Example:

Determine whether $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ is diagonalizable?

Solution:

The eigen values of A are -1 and 5. Since the eigen values are distinct their respective eigen vectors,

$$X_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{and} \quad X_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Are linearly independent, hence the matrix is diagonalizable. We can choose either

$$M = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} \text{ or } M = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

Making the first choice, we obtain

$$D = M^{-1}AM = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}$$

Making the second choice, wee obtain

$$D = M^{-1}AM = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$$

It illustrates a point we made previously, that neither M nor D is unique. However, the column of M must still correspond to the column of D that is once M is chosen that D is uniquely determined.

For example, if we choose $M = [X_2\, X_1\, X_3\, \ldots . X_n]$

Then, D must be,

$$\begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$$

While if we choose $M = [X_n\, X_{n-1} X_{n-2}\, \ldots . X_1]$ then D must be $\begin{bmatrix} \lambda_n & 0 & \cdots & 0 \\ 0 & \lambda_{n-1} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_1 \end{bmatrix}$

2.3 <u>Functions of Matrices – Diagonalizable Matrices</u>

2.3.1. Theorem

If $f(A)$ is well defined for a square matrix A and if A is diagonalizable then

$$f(A) = Mf(D)M^{-1} \qquad \text{......... (6)}$$

We can develop a simple procedure for computing functions of a diagonalizable matrix

$$D = \begin{pmatrix} \lambda_1 & & & \bigcirc \\ & \lambda_2 & & \\ & & \ddots & \\ \bigcirc & & & \lambda_n \end{pmatrix} \text{.....(7)} \qquad \text{then } D^m = \begin{pmatrix} \lambda_1^m & & & \bigcirc \\ & \lambda_2^m & & \\ \bigcirc & & \ddots & \\ & & & \lambda_n^m \end{pmatrix} \text{........(8)}$$

$$P_k(D) = \begin{pmatrix} P_k(\lambda_1) & & & \bigcirc \\ & P_k(\lambda_2) & & \\ \bigcirc & & \ddots & \\ & & & P_k(\lambda_n) \end{pmatrix} \text{......(9)} \qquad \text{and} \quad e^D = \begin{pmatrix} e^{\lambda_1} & & & \bigcirc \\ & e^{\lambda_2} & & \\ \bigcirc & & \ddots & \\ & & & e^{\lambda_n} \end{pmatrix} \text{......(10)}$$

Now assume that a matrix A is diagonalizable. Then it follows from (5) that $A = MDM^{-1}$

1) Thus, $A^2 = A.A = (MDM^{-1})(MDM^{-1})$

$$= (MD)(M^{-1}M)(DM^{-1})$$
$$= (MD)I(DM^{-1})$$
$$= MD^2M^{-1}$$

$$A^3 = A.A.A = (MDM^{-1})(MDM^{-1})(MDM^{-1})$$

$$= (MD)(M^{-1}M)(D)(M^{-1}M)(DM^{-1})$$

$$= (MD)IDIDM^{-1}$$

$$= (MD)D(DM^{-1}) = MD^3M^{-1}$$

And, in general, $A^n = MD^nM^{-1}$ .... (11)

Therefore, to obtain any power of a diagonalizable matrix A, we need only compute D, to that power, pre-multiply D^n by M and post multiply the result by M^{-1}.

2) A simplified expression for $P_k(A)$ where $P_k(x)$ is a k^{th} degree polynomial in x. for instance, suppose that $P_5(X) = 3X^2 + 2X^2 + 4$ & $P_5(A)$ is to be calculated. Making repeating use of (11), we have

$$P_5(A) = 5A^5 - 3A^3 + 2A^2 + 4I$$
$$= 5MD^5M^{-1} - 3MD^3M^{-1} + 2MD^2M^{-1} + 4MIM^{-1}$$
$$= M[5D^5 - 3D^3 + 2D^2 + 4I]M^{-1}$$
$$= MP_5\,(5)M^{-1}$$

Thus, to calculate $P_5(A)$, we need to compute only $P_5(D)$, pre-multiply this result by M^{-1}. If $P_k(A)$ is any k^{th} degree polynomial of A, then

$$P_k(A) = MP_k(D)M^{-1} \qquad \text{……… (12)}$$

3) The function of greatest interest is the exponential. If a matrix A is diagonalizable, to obtain a useful representation of e^A. In particular,

$$e^A = \sum_{k=0}^{\infty} \frac{A^k}{k!} = \sum_{k=0}^{\infty} \frac{(MDM^{-1})^k}{k!}$$

$$= M\left(\sum_{k=0}^{\infty} \frac{D^k}{k!}\right)M^{-1} = Me^DM^{-1} \qquad \text{……. (13)}$$

Thus, to calculate e^D and then pre multiply and post multiply by M^{-1} respectively.

Example: find Cos(A) for $A = \begin{bmatrix} 4\pi & 2\pi & 0 \\ -\pi & \pi & 0 \\ 3\pi & -2\pi & \pi \end{bmatrix}$

Solution

The eigen values of A are π,2π,3π, hence

$$D = \begin{bmatrix} \pi & 0 & 0 \\ 0 & 2\pi & 0 \\ 0 & 0 & 3\pi \end{bmatrix}$$

An appropriate M is found to be $M = \begin{bmatrix} 0 & 1 & -2 \\ 0 & -1 & 1 \\ 1 & 5 & -4 \end{bmatrix}$

$$\cos(A) = M\cos(D)M^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ 0 & -1 & 1 \\ 1 & 5 & -4 \end{bmatrix} \begin{bmatrix} \cos\pi & 0 & 0 \\ 0 & \cos\pi & 0 \\ 0 & 0 & \cos\pi \end{bmatrix} \begin{bmatrix} 1 & 6 & 1 \\ -1 & -2 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -4 & 0 \\ 2 & 3 & 0 \\ -10 & -20 & -1 \end{bmatrix}$$

2.4 Generalized Eigenvectors

Definition: A vector X_m is a generalized eigen vector of type m corresponding to the marix A and eigen value $\boldsymbol{\lambda}$, if

$$(A - \lambda I)^m X_m = 0 \quad \text{but} \quad (A - \lambda I)^{m-1} X_m \neq 0$$

For example, if $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ then $X_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

is a generalized eigen vector of type 3 corresponding to $\boldsymbol{\lambda}$=2. Since

$$(A - 2I)^3 X_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{but,}$$

$$(A - 2I)^2 X_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \neq 0$$

Also, $X_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

is a generalized eigen vector of type 2 corresponding to $\boldsymbol{\lambda}$= 2, since

$$(A - 2I)^2 X_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

But, $(A - 2I)^1 X_2 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \neq 0$

Furthermore, $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

is a generalized eigen vector of type 1 corresponding to $\boldsymbol{\lambda}$=2.

Since,

$(A-2I)^1X_1=0$ but $(A-2I)^0X_1=IX_1=X_1\neq 0$

2.4.1 Theorem

X_j is a generalized eigen vector of type j corresponding to the eigen value $\boldsymbol{\lambda}$.

Proof:

Since X_m is a generalized eigen vector of type m, $(A-\lambda I)^mX_m=0$ and

$(A-\lambda I)^{m-1}X_m\neq 0$

thus, we find that

$$(A-\lambda I)^jX_j=(A-\lambda I)^j(A-\lambda I)^{m-j}X_m=(A-\lambda I)^mX_m=0$$

and

$$(A-\lambda I)^{j-1}X_j=(A-\lambda I)^{j-1}(A-\lambda I)^{m-j}X_m=(A-\lambda I)^{m-1}X_m\neq 0$$

which together imply theorem.

We have found a generalized eigen vector of type m, it is simple to obtain a generalized eigen vector of any type less than m.

2.5 Chains

Definitions: let X_m be a generalized eigen vector of type m corresponding to the matrix A and the eigen value $\boldsymbol{\lambda}$. The chain generated by X_m is a set of vectors

$\{X_m, X_{m-1}, \dots . X_1\}$ given by

$$\begin{aligned} X_{m-1} &= (A-\lambda I)X_m \\ X_{m-2} &= (A-\lambda I)^2X_m=(A-\lambda I)X_{m-1} \\ X_{m-3} &= (A-\lambda I)^3X_m=(A-\lambda I)X_{m-2} \qquad \dots\dots.. (14) \\ &\;\;\vdots \\ X_1 &= (A-\lambda I)^{m-1}X_m=(A-\lambda I)X_2 \end{aligned}$$

Thus, in general,

$$X_j = (A - \lambda I)^{m-j} X_m = (A - \lambda I) X_{j+1} \quad (\mathrm{j} = 1,2,3, \dots, \mathrm{m} - 1) \qquad \text{.........(15)}$$

2.5.1 Theorem

A chain is a linear independent set of vectors.

Proof:

Let $\{X_m, X_{m-1}, \dots . X_1\}$ be a chain generated from X_m, a generalized eigen vector of type m corresponding to the eigen value $\boldsymbol{\lambda}$ of a matrix A, and consider the matrix equation

$$C_m X_m + C_{m-1} X_{m-1} + \cdots + C_1 X_1 = 0 \qquad \text{........... (16)}$$

In order to prove that this chain is a linearly independent set, we must show that the only constants satisfying in the above equation are $\mathrm{C_m} = \mathrm{C_{m-1}} = \cdots = \mathrm{C_1} = 0$.

Multiply the equation by $(A - \lambda I)^{m-1}$ and note that for $j = 1,2,3, \dots, m - 1$

$$(A - \lambda I)^{m-1} C_j X_j = C_j (A - \lambda I)^{m-j-1} (A - \lambda I)^j X_j = C_j (A - \lambda I)^{m-j-1} * 0$$

(Since, X_j is a generalized eigen vector of type j)

$$(A - \lambda I)^{m-1} C_j X_j = 0$$

Thus, (16) becomes $C_m (A - \lambda I)^{m-1} X_m = 0$. However, since X_m is a generalized eigen vector of type m, $(A - \lambda I)^m X_m \neq 0$. From which it follows that $C_m = 0$, substituting $\mathrm{C}_m = 0$, into (16) and then multiplying (16) by $(A - \lambda I)^{m-2}$, we find by similar reasoning that $C_{m-1} = 0$. Continuing this process, we finally obtain $C_m = C_{m-1} = \cdots = C_1 = 0$, which implies that the chain is linearly independent.

CHAPTER 3

JORDAN CANONICAL FORMS

3.1 Canonical basis

3.1.1 Theorem

Every $n \times n$ matrix A possesses n linearly independent generalized eigen vectors, henceforth abbreviated 'liges' generalized eigen vectors corresponding to distinct eigen values are linearly independent. If λ is an eigen value of A of multiplicity V, then A will have V liges corresponding to λ.

Definition:

A set of n liges is a canonical basis for a $n \times n$ matrix if the set is composed entirely of chains.

Thus, once we have determined that a generalized eigen vector of type m is in a canonical basis it follows that the $m-1$ vectors $X_{m-1}, X_{m-2}, \dots, X_1$ that are in the chain generated by X_m given by (14) are also in the canonical basis.

Let λ_i be an eigen value of A of multiplicity V. First, find the rank of the matrices $(A-\lambda_i I)$, $(A-\lambda_i I)^2, \dots, (A-\lambda_i I)^m$.The integer m is determined to be the first integer for which $(A-\lambda_i I)^m$ has rank $(n-V)$, n being the number of rows or columns of A that is, A is $n \times n$.

Example:

Determine $'m'$ corresponding to λ_i=2 for A=$\begin{pmatrix} 2 & 1 & -1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}$

Solution:

$n = 6$ and the eigen value $\lambda_i = 2$ has multiplicity $V = 5$, hence $n - V = 1$

consider the matrix $A - 2I$, has the rank 4.

$$A-2I=\begin{pmatrix} 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix} \quad \ldots..(17)$$

Consider the matrix

$$(A-2I)^2=\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix} \quad \ldots..(18)$$

and

$$(A-2I)^3=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 8 \end{pmatrix} \quad \ldots.(19)$$

has rank 1 which is $n-V$. Therefore, corresponding to $\lambda_i = 2$, we have $m = 3$. Now define,

$$\boldsymbol{\rho_k = r(A-\lambda_i I)^{k-1} - r\,(A-\lambda_i I)^k \qquad k = 1, 2, 3, \ldots, m} \qquad \ldots\ldots (20)$$

ρ_k designates the number of liges of type k corresponding to the Eigen value λ_i that will appear in a canonical basis for A. Note that $r(A-\lambda_i\, I)^0 = r(I) = n$

Example:

Find a canonical basis for $A = \begin{pmatrix} 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Solution:

A is a 4×4 and $\lambda_i = 1$ is an eigen value of multiplicity 4, hence $n = 4, V = 4\ and\ n - V = 0$

$(A - 1I) =$

$$\begin{pmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Has type 2,

and $(A - 1I)^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Has type $0 = n - V$. Thus $m = 2$,

$\rho_2 = r\ (A - 1I) - r\ (A - 1I)^2 = 2 - 0 = 2$ and

$\rho_1 = r(A - 1I)^0 - r(A - 1I)^1 = 4 - 2 = 2$;

hence a canonical basis for A will have 2 liges of type 2 and 2 liges of type 1. In order for a vector to be a $[w \quad x \quad y \quad z]^T$ to be generalized eigen vector of type 2, either x or z must be non zero and w and y arbitrary. If we choose $x = 1, w = y = z = 0$ and the choose $z = 1, w = x = y = 0$ we obtain two liges of type 2 to be

$$X_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad and\ Y_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Note that we could have chosen w, x, y and z and z in such a manner as to generate four linearly independent generalized eigen vectors of type 2. The vectors

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 \\ 1 \\ 1` \\ 1 \end{bmatrix}$$

Together with X_2 and Y_2 form such a set. Thus, we immediately have found a set of four liges corresponding to $\lambda_1 = 1$. This set, however is not a canonical basis for A, since it is not composed of chains. In order to obtain a canonical basis for A, we use only two of these vectors (in particularly X_2 and Y_2) and form chains from them. Using (14) we obtain the two liges of type 1 to be

$$X_1 = (A - I)X_2 = [\,1 \quad 0 \quad 0 \quad 0]^T \quad \text{and} \quad Y_1 = (A - I)Y_2 = [-1 \quad 0 \quad 1 \quad 0]^T$$

Thus a canonical basis for A is $\{X_1, X_2, Y_1, Y_2\}$, which consists of the two chains $\{X_2, X_1\}$ and $\{Y_2, Y_1\}$ each containing two vectors.

3.2 Jordan canonical form

In this section we will show that every matrix is similar to an "almost diagonal" matrix, or in more precise terminology, a matrix in Jordan canonical form. We start by defining a square matrix S_k(k represents some positive integer and has no direct bearing on the order of S_k), given by

$$S_k = \begin{pmatrix} \lambda_k & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & \lambda_k & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & \lambda_k & 1 & \dots & 0 & 0 \\ . & . & . & . & \dots & . & . \\ . & . & . & . & \dots & . & . \\ . & . & . & . & \dots & . & . \\ 0 & 0 & 0 & 0 & \dots & \lambda_k & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 & \lambda_k \end{pmatrix} \qquad \dots\dots\dots..(21)$$

Thus S_k is a matrix that has all of its diagonal elements equal to λ_k ,all of its super diagonal elements (that is all elements directly above the diagonal elements) equal to 1, and all of its other elements equal to zero. This matrix is known as Jordan block (cell).

Definition 1

A square matrix A is in Jordan canonical form if it is a diagonal matrix or can be expressed in either one of the following two partitioned diagonal forms:

$$\begin{pmatrix} D & & & & & & \bigcirc \\ & S_1 & & & & & \\ & & S_2 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ \bigcirc & & & & & & S_r \end{pmatrix} \quad \text{or} \qquad \ldots\ldots..(22)$$

$$\begin{pmatrix} S_1 & & & & & \bigcirc \\ & S_2 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ \bigcirc & & & & & S_r \end{pmatrix} \qquad \ldots\ldots(23)$$

Here D is a diagonal matrix

Example

Consider the following matrix $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ ……..(24)

It is in Jordan canonical form since it can be written $\begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}$

where, $S_1 = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ and $S_2 = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$

Consider $\begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ is not Jordan Canonical Form, Because of elements on super diagonal are 2.

Note that a matrix in JCF has nonzero elements only on the main diagonal and super diagonal and that the elements on the super diagonal are restricted to be either zero or one. In particular, a diagonal matrix is a matrix in JCF that has all its super diagonal elements equal to zero.

Definition 2

Let A be a $n \times\times n$ matrix. A generalized modal matrix M for A is an $n \times n$ matrix whose columns, considered as vectors, form a canonical basis for A and appear in the first column of M according to the following rule:

(M_1) All chains consisting to one vector that is, one vector in length appear in the first columns of M

(M_2) All vectors of the same chain appear together in adjacent columns of M.

(M_3) Each chain appears in M in order of increasing type that is the generalized eigen vector of type 2 of the same chain, which appears before the generalized eigen vector of type 3 of the same chain etc

Example:

find a generalized modal matrix M corresponding to the

$$A = \begin{pmatrix} 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Solution:

We found that a canonical basis for A has two chains consisting of two vectors a piece $\{X_2, X_1\}$ and $\{Y_2, Y_1\}$. Since this canonical basis has no chain consisting of one vector M_1 does not apply.

From M_2 we assign either X_2 and X_1 to the first two columns of M and, Y_2 and Y_1 to the last two columns of M or alternatively, Y_2 and Y_1 to the first two columns of M and X_2 and X_1 to the last two columns of M. We cannot, however, define $M = [X_1 \ X_2 \ Y_1 \ Y_2]$ since this alignment would split the $\{X_2, X_1\}$ chain and violate M_2. Due to (M_3), X_1 must precede X_2 and Y_1 must precede Y_2; hence

$$M = [X_1 \ X_2 \ Y_1 \ Y_2] = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$M = [Y_1 \ Y_2 \ X_1 \ X_2] = \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

It is shows that M is not unique. The important fact, however is that for any arbitrary $n \times n$ matrix A, there does exist at least one generalized modal matrix M corresponding to it.. Furthermore, since the columns of M considered as vectors form a linearly independent set, it follows that the column rank of M is n, the rank of M is n, the determinant of M is nonzero, and M is invertible (that is M^{-1} exists)

3.2.1 Theorem

Every $n \times n$ matrix A is similar to a matrix in Jordan canonical form.

Proof:

Now let A represent any $n \times n$ matrix and let M be a generalized modal matrix for A.

$$AM = MJ$$

We know prove the validity of this equation, which states that if A is an arbitrary nxn matrix and M is a generalized modal matrix for A, then $AM = MJ$ where J is a matrix in JCF. In particular case can be extended easily to cover any arbitrary case.

We assume that the canonical basis used to form M consisted of one chain containing 3 vectors $\{X_3, X_2, X_1\}$ and one chain containing one vector $\{Y_1\}$ The three element chain corresponds to the eigen value λ_1 while Y, corresponds to the eigen value λ_2; λ_1 and λ_2 can be equal or distinct.

Since X_1 and Y_1 are both generalized eigen vectors of type 1, they are themselves eigen vectors, hence $AX_1 = \lambda_1 X_1$ and $AY_1 = \lambda_2 Y_1$. Furthermore, since X_2 and X_1 belong to the chain generated by X_3 it follows that

$$X_2 = AX_3 - \lambda_1 X_3$$

$$X_1 = AX_2 - \lambda_1 X_2$$

or, by a rearrangement of terms,

$$AX_3 = \lambda_1 X_3 + X_2$$

$$AX_2 = \lambda_1 X_2 + X_1$$

Thus,

$$AM = A[Y_1\ X_1\ X_2\ X_3] = [AY_1\ \ AX_1\ \ AX_2\ \ AX_3]$$

$$= [\lambda_2 Y_1 \quad \lambda_1 X_1 \quad \lambda_1 X_2 + X_1 \quad \lambda_1 X_3 + X_2]$$

$$= [Y_1 \quad X_1 \quad X_2 \quad X_3] \begin{pmatrix} \lambda_2 & 0 & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 \\ 0 & 0 & \lambda_1 & 1 \\ 0 & 0 & 0 & \lambda_1 \end{pmatrix}$$

Defining $J = \begin{pmatrix} \lambda_2 & 0 & 0 & 0 \\ 0 & \lambda_2 & 1 & 0 \\ 0 & 0 & \lambda_2 & 1 \\ 0 & 0 & 0 & \lambda_2 \end{pmatrix}$

.

Which is a matrix in JCF, we obtain $AM = MJ$, the desired result. By either pre multiplying or post multiplying by $M^{-1,}$ obtain either

$$J = M^{-1}AM \qquad or \qquad A = MJM^{-1}$$

Example:

Find a matrix in JCF that is similar to A = $\begin{pmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{pmatrix}$

Solution:

The characteristic of A is $(\lambda - 2)^3 = 0$ Hence λ = 2 is an eigen value of multiplicity three Following the procedure we find that

$$r(A - 2I) = 1 \quad \text{and} \quad r(A - 2I)^2 = 0 = n - V$$

Thus, $\rho_2 = 1$ and $\rho_1 = 2$, which implies that a canonical basis for A will contain one linearly generalized eigen values of type2 and two liges of type1 or equivalently, one chain of two vectors $\{X_2, X_1\}$ and one chain of one vector $\{Y_1\}$. Designating $M = \{Y_1, X_1, X_2\}$,

we find that

M = $\begin{pmatrix} 2 & 2 & 0 \\ 1 & 3 & 0 \\ 0 & -4 & 1 \end{pmatrix}$ thus $M^{-1} = \frac{1}{4}\begin{pmatrix} 3 & -2 & 0 \\ -1 & 2 & 0 \\ -4 & 8 & 4 \end{pmatrix}$

and

$J = M^{-1}AM = \frac{1}{4}\begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 4 \\ 0 & 0 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

It is possible to construct a J matrix in JCF associated with a given nxn matrix A.Just from knowledge of the composition of M.

Each complete chain of more than one vector in length that goes into composing M will give rise to a S_k sub matrix in J.Thus, for example if a canonical basis for A contains a chain of three elements corresponding to the eigen value λ, the matrix J that is similar to A must contain a sub matrix

$$S_k = \begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix}$$

The order of S_k is identical to the length of the chain. The chain consisting of only one vector gives rise collectively to the D sub matrix in J. Thus, if there were no chains of one vector in a canonical basis for A, then J would contain no D sub matrix while if a canonical basis for A contained 4 one vector chains, then J would contain a D sub matrix of order 4×4 .In this latter case, the elements on the main diagonal of D would be the eigen values corresponding to the one element chains.

Finally, by rearranging the order in which whole chains are placed in to M, we merely rearrange the order in which the corresponding S_k, sub matrices appear in J. For example, suppose that the characteristic equation of A is $(\lambda-1)(\lambda-2)(\lambda-3)^2(\lambda-4)^2$.

Furthermore, assume that $\lambda = 3$ gives rise to the chain $\{Z_1\ Z_2\}, \lambda = 4$ gives rise to the chain $\{W_1\ \ W_2\}$ and the eigen values $\lambda = 1\ \ and\ \lambda = 2$ correspond respectively to the eigen vectors X_1 and Y_1. Then, if we choose

$M = [X_1\quad Y_1\quad Z_1\quad Z_2\quad W_1\quad W_2]$ it will follow that

$$M^{-1}AM = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

While if we pick $M = [Y_1\quad X_1\quad W_1\quad W_2\quad Z_1\quad Z_2]$ it will follow that

$$M^{-1}AM = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

3.3 Number of linearly independent eigen vector in JCF

Let a $n \times n$ matrix in JCF has m 1's in super diagonal. Then the number of linearly independent eigen vectors = $n - m$

Thus, for example in J = $\begin{bmatrix} J_1 & \vdots & 0 \\ 0 & \vdots & 5 \end{bmatrix}$ where J_1 is a 4×4 Jordan block.

For example

i) If $J_1 = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$ then number of linearly independent eigen vectors in $J = n - m = 5 - 0 = 5$

ii) If $J_1 = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$ then number of linearly independent eigen vectors in $J = n - m = 5 - 2 = 3$

Geometric multiplicities m of an eigen value λ is m linearly independent eigen vectors associated with 1 and evidently it is the number of Jordan blocks associated with 1. Thus geometric multiplicities of J_1 in (i) and (ii) are 4 and 3 respectively for λ=2.

Eigen vectors in JCF are identified from absence of 1's above the corresponding eigen value while generalized eigen vectors are identified from the presence of 1's above the corresponding eigen values

3.4 JCF from rank

3.4.1 Sizes of blocks determined from the rank of powers $(J - \lambda I)$

Let J be the direct sum corresponding to the Eigen value λ. If $(J - \lambda I)^{k_1} = 0$ for some smallest integer k_1, the size of largest block k_1 and k_2 is called **index** of Eigen value λ. The rank of $(J - \lambda I)^{k_1 - 1}$ is the number of blocks of order k_1. The rank of $(J - \lambda I)^{k_1 - 2}$ = [(number of blocks of order k_1) +(number of blocks of size (k_1 - 1) | and so forth.

For example, if

J =

$$\begin{pmatrix} \begin{array}{ccc|cc|cc|c} 3 & 1 & 0 & & & & & \\ 0 & 3 & 1 & & & & O & \\ 0 & 0 & 3 & & & & & \\ \hline & & & 3 & 1 & & & \\ & & & 0 & 3 & & & \\ \hline & & & & & 3 & 1 & \\ & O & & & & 0 & 3 & \\ \hline & & & & & & & 3 \end{array} \end{pmatrix} \quad \text{then } J - 3I = \begin{pmatrix} \begin{array}{ccc|cc|cc|c} 0 & 1 & 0 & & & & & \\ 0 & 0 & 1 & & & & O & \\ 0 & 0 & 0 & & & & & \\ \hline & & & 0 & 1 & & & \\ & & & 0 & 0 & & & \\ \hline & & & & & 0 & 1 & \\ & O & & & & 0 & 0 & \\ \hline & & & & & & & 0 \end{array} \end{pmatrix}$$

$$(J-3I)^2 = \begin{pmatrix} \begin{array}{ccc|cc|cc|c} 0 & 0 & 1 & & & & & \\ 0 & 0 & 0 & & & & O & \\ 0 & 0 & 0 & & & & & \\ \hline & & & 0 & 0 & & & \\ & & & 0 & 0 & & & \\ \hline & & & & & 0 & 0 & \\ & O & & & & 0 & 0 & \\ \hline & & & & & & & 0 \end{array} \end{pmatrix} \quad \text{and } (J - 3I)^3 = 0$$

Thus $(J-3I)^3 = 0$ implies that $k_1 = 3$ is the size of largest block. Then

Rank $(J-3I)^2 = 1 =$ the number of block size 3

Rank $(J-3I) = 4 =$2 (number of block of size 3) + (number of blocks of size (3 - 1))

= 2.1 + 2 = 4 = total number of blocks in J.

3.4.2 <u>Algorithm</u>

Step 1: Find all distinct eigen values of an $n \times n$ matrix A.

Step 2: For each distinct eigen value λ_i compute $(A - \lambda_i I)^k, k = 1,2,3, \ldots, n$ until

Step 3: Compute the rank of matrices in step 2

Step 4: Decide the order and the numbers of blocks in JCF of A corresponding to each other eigen value λ_i

Step 5: Arrange the blocks from largest at the left top down to smallest gradually to the right bottom with equal blocks of same eigen value in consecutive order.

Example

Reduce to the matrix $\begin{pmatrix} 5 & -1 & 1 & 1 & 0 & 0 \\ 1 & 3 & -1 & -1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 & -1 & -1 \\ 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{pmatrix}$ to JCF

Solution :

Step 1: The characteristic equation is $(\lambda - 4)^5 (\lambda - 2) = 0$

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 4 \quad \text{and} \quad \lambda_6 = 2$$

Step 2: Now $(A - 4I) = \begin{pmatrix} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$

$(A - 4I) = \begin{pmatrix} 0 & 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & -2 & 2 \end{pmatrix}$ and $(A - 4I)^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & 4 \\ 0 & 0 & 0 & 0 & 4 & -4 \end{pmatrix}$

Higher power >3 of $(A - 4I)$ will resemble $(A - 4I)^3$ expect the different entries in 2×2 block in the lower corner. This implies that any vector annihilated by some power of $(A - 4I)$ will be annihilated by $(A - 4I)^3$. Thus, the index of eigen value 4 is 3 and so the size of largest block = 3 with eigen value λ= 4.

Step 3: Now, by row reduction, it can be easily seen that rank $(A-41)^3=1$,

rank $(A-41)^2=2$ and rank$(A-41)=3$

Step 4 : Since rank $(A - 4I)^{4-1} = 1$, so (the number of blocks of order 3) = 1.Since λ = 4 occurs 5 times and largest block contains 3 eigen values $\lambda = 4$.

Again, $rank(A - 4I)^{3-2} = rank(A - 4I) = 3$.

Then,

2(number of blocks of size 3) + (number of blocks of size (3 - 1) =3

Or

(number of blocks of size 2 with eigen value λ= 4) = 3 - 2.1 = 1

Finally, λ = 2 is distinct. So, there must be one block with eigen value λ = 2

Step 5 : Hence the JCF is J =

$$J = \left(\begin{array}{ccc|cc|c} 4 & 1 & 0 & & & \\ 0 & 4 & 1 & & O & \\ 0 & 0 & 4 & & & \\ \hline & & & 4 & 1 & \\ & O & & 0 & 4 & \\ \hline & & & & & 2 \end{array}\right)$$

CHAPTER 4

APPLICATION OF JORDAN CANONICAL FORMS

4.1 Functions Of Matrices – General Case

By using $A = MJM^{-1}$, we can generalize the results of section 2.3 and develop a method for computing functions of non-diagonalizable matrices we begin by directing our attention to those matrices that are already in JCF.

Consider any arbitrary n×n matrix J in the JCF.

$$J = \begin{bmatrix} D & & & & \bigcirc \\ & S_1 & & & \\ & & S_2 & & \\ & & & \ddots & \\ \bigcirc & & & & S_r \end{bmatrix}$$

$$J^2 = \begin{bmatrix} D & & & & \bigcirc \\ & S_1 & & & \\ & & S_2 & & \\ & & & \ddots & \\ \bigcirc & & & & S_r \end{bmatrix} \begin{bmatrix} D & & & & \bigcirc \\ & S_1 & & & \\ & & S_2 & & \\ & & & \ddots & \\ \bigcirc & & & & S_r \end{bmatrix} = \begin{bmatrix} D^2 & & & & \bigcirc \\ & S_1^2 & & & \\ & & S_2^2 & & \\ & & & \ddots & \\ \bigcirc & & & & S_r^2 \end{bmatrix}$$

$$J^3 = J^2.J = \begin{bmatrix} D^3 & & & & \bigcirc \\ & S_1^3 & & & \\ & & S_2^3 & & \\ & & & \ddots & \\ \bigcirc & & & & S_r^3 \end{bmatrix}$$

And in general,

$$J^n = \begin{bmatrix} D^n & & & & \bigcirc \\ & S_1^n & & & \\ & & S_2^n & & \\ & & & \ddots & \\ \bigcirc & & & & S_r^n \end{bmatrix}$$

Furthermore, if $f(z)$ is a well-defined function for J or equivalently $D, S_1, S_2, \dots, S_r$ then,

$$f(J) = \begin{bmatrix} f(D) & & & & \bigcirc \\ & f(S_1) & & & \\ & & f(S_2) & & \\ & & & \ddots & \\ \bigcirc & & & & f(S_r) \end{bmatrix} \qquad \dots\dots\dots\dots.(25)$$

Since $f(D)$ has already been determined in section 2.3, we only need develop a method for calculating $f(S_k)$ in order to have $f(J)$ determined completely. We have $(P+1) \times (P+1)$ matrix S_k defined by

$$S_k = \begin{bmatrix} \lambda_k & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & \lambda_k & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & \lambda_k & 1 & \dots & 0 & 0 \\ . & . & . & . & \dots & . & . \\ . & . & . & . & \dots & . & . \\ . & . & . & . & \dots & . & . \\ 0 & 0 & 0 & 0 & \dots & \lambda_k & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 & \lambda_k \end{bmatrix}$$

It can be shown that,

$$f(S_k) = \begin{bmatrix} f(\lambda_k) & \frac{f'(\lambda_k)}{1!} & \frac{f''(\lambda_k)}{2!} & \dots & \frac{f^{(p)}(\lambda_k)}{p!} \\ 0 & f(\lambda_k) & \frac{f'(\lambda_k)}{1!} & \dots & \frac{f^{(p-1)}(\lambda_k)}{(p-1)!} \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ 0 & 0 & 0 & \dots & 0 & f(\lambda_k) \end{bmatrix} \qquad \dots\dots\dots (26)$$

Caution: the derivatives in f (S_k) are taken with respect to $\boldsymbol{\lambda}_k$. For instance, if $\lambda_k = 3t$ and $f(\lambda_k) = e^{\lambda_k} = e^{3t}$, then $f''(\lambda_k) \neq 9e^{3t}$, but rather e^{3t}. That is, the derivative must be taken with respect to $\lambda_k = 3t$ and not with respect to t. Perhaps the safest way to make the necessary computation in $f(S_k)$ without incurring an error is to first keep $f(\lambda_k)$ in terms of $\boldsymbol{\lambda}_k$ not substitute a numerical value for $\boldsymbol{\lambda}_k$ such as 3t, then take the derivative of f($\boldsymbol{\lambda}_k$) with respect to $\boldsymbol{\lambda}_k$ (the second derivative of e^{λ_k} with respect to $\boldsymbol{\lambda}_k$ is e^{λ_k} .), and finally, as the last step, substitute in the correct value for $\boldsymbol{\lambda}_k$ where needed.

The Jordan Canonical Form of a matrix is a very important concept from Linear Algebra. One of its most important applications lies in the solving systems of ordinary differential equations. Fundamental Theorem for Linear Systems can also be proved by the help of JCF.

Example:

Find e^{S_k} if $S_k = \begin{bmatrix} 2t & 1 & 0 \\ 0 & 2t & 1 \\ 0 & 0 & 2t \end{bmatrix}$

Solution:

In this case $\lambda_k = 2t, f(S_k) = e^{S_k}$ and $f(\lambda_k) = e^{\lambda_k}$

$$e^{\lambda_k} = f(S_k) = \begin{bmatrix} f(\lambda_k) & \frac{f'(\lambda_k)}{1!} & \frac{f''(\lambda_k)}{2!} \\ 0 & f(\lambda_k) & \frac{f'(\lambda_k)}{1!} \\ 0 & 0 & f(\lambda_k) \end{bmatrix} = \begin{bmatrix} e^{\lambda_k} & e^{\lambda_k} & \frac{e^{\lambda_k}}{2} \\ 0 & e^{\lambda_k} & e^{\lambda_k} \\ 0 & 0 & e^{\lambda_k} \end{bmatrix}$$

$$= \begin{bmatrix} e^{2t} & e^{2t} & \frac{e^{2t}}{2} \\ 0 & e^{2t} & e^{2t} \\ 0 & 0 & e^{2t} \end{bmatrix} = e^{2t} \begin{bmatrix} 1 & 1 & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Now let A be a n×n matrix. We know from the previous section that there exists a matrix J in Jordan canonical form and an invertible generalized modal matrix M such that

$$A = MJM^{-1}$$

By an analysis identical to that used in section 2.3 to obtain (7), (11) and (12) we have

$$f(A) = Mf(J)M^{-1}$$

Providing, of course, that f(A) is well defined. Thus, $f(A)$ is obtained simply by first calculating $f(J)$, which in view of (25) can be done quite easily, then pre multiplying $f(J)$ by M, and finally post multiplying this result by M^{-1} .

CONCLUSION

The Jordan canonical form has many uses in linear and abstract algebra. When it can happen, diagonalization is quite useful, and it is good to know that we can always get something very close to diagonal form with Jordan canonical form. Also, the Jordan canonical form of a matrix is of use in solving differential equations.

Canonical (standard or normal) forms of matrices are of different types. They are simpler than original matrices and are used in variety of application in other areas of mathematics and science for the study of required information of original matrices. These are obtained by post multiplication and pre multiplication of unitary orthogonal and modal matrices etc. One of them is discussed in this project that is Jordan canonical form.

An n square Jordan canonical form, J is a diagonal block partitioned matrix in which each of $m_i \times m_i$, blocks have Eigen value λ_i, of multiplicity m_i, and first super diagonal of each block consists of 1 unless a Jordan block is a diagonal one while rest of the elements are zero. The block are the Jordan cell or blocks. The matrix J is known as Jordan matrix. It is so named after the name of French mathematician, C. Jordan.

Throughout this project we discussed about similar matrices, diagonalizable matrix, Jordan block and Jordan canonical forms its definition and Jordan canonical form from rank - an algorithm at last application of Jordan canonical form.

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GRAPH COLOURING

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INTRODUCTION

An irreducible polynomial is a polynomial that cannot be factored into nontrivial factors over the same field. This means that an irreducible polynomial cannot be expressed as the product of two or more polynomials that are not constants, over the same field. Irreducible polynomials play an important role in various areas of mathematics, including algebraic geometry, number theory, and coding theory.

This project covers the basics of rings, fields, integral domain, what irreducible polynomials are, and the various methods to find if a polynomial is irreducible or not which are, Eisenstein's criterion, the p test, and cyclotomic polynomials.

OUTLINE OF THE PROJECT

The topic we are considering for this project is Irreducible Polynomials, which is a fundamental concept in algebra and number theory.

Chapter 1 covers the basics of ring theory and gives an outlook of the field and integral domain. Since a polynomial is defined on either of these it is necessary to know about these concepts.

Chapter 2 covers polynomial rings, division algorithm and factor theorem. The irreducibility of a polynomial revolves around the fact that the polynomial cannot be split into its factors which makes it essential to know about the theorems.

Chapter 3 explains what irreducible polynomials are and also covers a few examples.

Chapter 4 discusses three methods that help us to check if a polynomial is irreducible or not.

GRAPH COLOURING

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Athul U S (CCAUSMT055)

CONTENTS

JORDAN CANONICAL FORM

Project report submitted to Christ College (Autonomous) in partial fulfilment of requirement for the award of the B.Sc. Degree Programme in Mathematics

By

Devika Linson (CCAUSMT056)

Department of Mathematics (Unaided)

Christ College (Autonomous)

Irinjalakuda

2023

CERTIFICATE

This is to certify that the project entitled "JORDAN CANONICAL FORM" submitted to the Department of Mathematics (Unaided) in partial fulfilment of the requirement for the award of the B.Sc. Degree programme in Mathematics, is a bonafide record of original research work done by Devika Linson (CCAUSMT056) during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda under my supervision and guidance during the year 2022-2023.

External Examiner:

Ms. Christina P J
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Department of Mathematics (Unaided)
Christ College (Autonomous) Irinjalakuda

Dr. JOJU K T
Coordinator
Department of Mathematics Unaided
Christ College (Autonomous) Irinjalakuda

Place: Irinjalakuda

Date:18-04-2023

DECLARATION

I hereby declare that the project work entitled “JORDAN CANONICAL FORM” submitted to the Christ College (Autonomous) in partial fulfilment of the requirement for the award of the B.Sc. Degree programme in Mathematics is a record of original project work done by me during the period of my study in the Department of Mathematics Unaided, Christ College (Autonomous),Irinjalakuda.

Devika Linson (CCAUSMT056)

Place: Irinjalakuda

Date: 18/04/2023

ACKNOWLEDGEMENT

I wish to acknowledge our indebtedness to a number of colleagues who made valuable efforts and suggestions to the content of this project. There are many individuals who have come together to make this project a reality. I greatly appreciate the inspiration, support and guidance of all those people who have been instrumental for making this project a success.

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Devika Linson (CCAUSMT056)

CONTENT

INTRODUCTION

Any linear transformation can be represented by its matrix representation. In an ideal situation, all linear operators can be represented by a diagonal matrix. However, in the real world, there exist many linear operators that are not diagonalizable. This gives rise to the need for developing a system to provide a beautiful matrix representation for a linear operator that is not diagonalizable.

Matrices are "natural" mathematical objects: they appear in connection with linear equations, linear transformations, and also in conjunction with bilinear and quadratic forms, which were important in geometry, analysis, number theory, and physics. Matrices as rectangular arrays of numbers appeared around 200 BC in Chinese mathematics, but there they were merely abbreviations for systems of linear equations. Matrices become important only when they are operated on added, subtracted, and especially multiplied; more important, when it is shown what use, they are to be put to.

Cayley advanced considerably the important idea of viewing matrices as constituting a symbolic algebra. In particular, his use of a single letter to represent a matrix was a significant step in the evolution of matrix algebra. But his papers of the 1850s were little noticed outside England until the 1880s. During the intervening years deep work on matrices (in one guise or another) was done on the continent, by Cauchy, Jacobi, Jordan, Weierstrass, and others. They created what may be called the spectral theory of matrices: their classification into types such as symmetric, orthogonal, and unitary; results on the nature of the Eigen values of the various types of matrices; and, above all, the theory of canonical forms for matrices the determination, among all matrices of a certain type, of those that are canonical in some sense. An important example is the Jordan canonical form, introduced by Weierstrass (and independently by Jordan), who showed that two matrices are similar if and only if they have the same Jordan canonical form.

IRREDUCIBLE POLYNOMIAL

Project report submitted to Christ College (Autonomous) in partial fulfillment of the requirement for the award of the B.Sc. Degree Programme in Mathematics

By

Devikrishna C R (CCAUSMT057)

Department of Mathematics (Unaided)

Christ College (Autonomous)

Irinjalakuda

2023

CERTIFICATE

This is to certify that the entitled “IRREDUCIBLE POLYNOMIAL” Submitted to the Department of Mathematics (Unaided) in partial fulfillment of the requirement for the award of the B.Sc. Degree programme in Mathematics is a bonafide record of original research work done by Devikrishna C R (CCAUSMT057) during the period of her study in the Department of Mathematics (Unaided), Christ College (Autonomous), Irinjalakuda under my supervision and guidance during the year 2022 – 2023.

Mr. Jomesh Jose	Dr. Joju K T
Adhoc Lecture	Coordinator
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Christ College (Autonomous),	Christ College (Autonomous),
Irinjalakuda	Irinjalakuda

External Examiner

Place: Irinjalakuda

Date: 18/04/2023

DECLARATION

I hereby declare that the project work entitled “IRREDUCIBLE POLYNOMIAL” submitted to Christ College (Autonomous) in partial fulfillment of the requirement for the award of the B.Sc. Degree programme in Mathematics is a record of original project work done by me during the period of our study in the Department of Mathematics (Unaided), Christ College (Autonomous), Irinjalakuda.

Devikrishna C R (CCAUSMT057)

Place: Irinjalakuda

Date: 18/04/2023

ACKNOWLEDGEMENT

I wish to acknowledge my indebtedness to a number of colleagues who made valuable efforts and suggestions for the content of this project. There are many individuals who have come together to make this project a reality. I greatly appreciate the inspiration, support and guidance of all those people who have been instrumental in making this project a success.

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Words cannot express the love and support we have received from my parents, whose encouragement has buoyed me up from the beginning till the end of this work.

Devikrishna C R (CCAUSMT057)

CONTENTS

INTRODUCTION

An irreducible polynomial is a polynomial that cannot be factored into nontrivial factors over the same field. This means that an irreducible polynomial cannot be expressed as the product of two or more polynomials that are not constants, over the same field. Irreducible polynomials play an important role in various areas of mathematics, including algebraic geometry, number theory, and coding theory.

This project covers the basics of rings, fields, integral domain, what irreducible polynomials are, and the various methods to find if a polynomial is irreducible or not which are, Eisenstein's criterion, the p test, and cyclotomic polynomials.

OUTLINE OF THE PROJECT

The topic we are considering for this project is Irreducible Polynomials, which is a fundamental concept in algebra and number theory.

Chapter 1 covers the basics of ring theory and gives an outlook of the field and integral domain. Since a polynomial is defined on either of these it is necessary to know about these concepts.

Chapter 2 covers polynomial rings, division algorithm and factor theorem. The irreducibility of a polynomial revolves around the fact that the polynomial cannot be split into its factors which makes it essential to know about the theorems.

Chapter 3 explains what irreducible polynomials are and also covers a few examples.

Chapter 4 discusses three methods that help us to check if a polynomial is irreducible or not.

CRYPTOGRAPHY

A project report submitted to Christ College (Autonomous) in partial fulfilment of requirement for the award of the B.Sc. Degree programme in Mathematics

By

Gokul Venu (CCAUSMT058)

DEPARTMENT OF MATHEMATICS

CHRIST COLLEGE (AUTONOMOUS)

IRINJALAKUDA

2023

CERTIFICATE

This is to certify that the project entitled "CRYPTOGRAPHY" submitted to the Department of Mathematics in partial fulfillment of the requirement for the award of the B.Sc. Degree programme in Mathematics, is a bonafide record of original research work done by Gokul Venu(CCAUSMT058) during the period of their study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda under my supervision and guidance during the year 2022-2023.

External Examiner**:**

Dr. JOJU K T
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Department of Mathematics (Unaided)
Christ College (Autonomous)
Irinjalakuda

Dr. JOJU K T

Associate Professor

Department Of Mathematics (Unaided)

Christ College Irinjalakuda

Place: Irinjalakuda

Date: 18-04-2023

DECLARATION

We hereby declare that the project work entitled "CRYPTOGRAPHY" submitted to the Christ College (Autonomous) in partial fulfillment of the requirement for the award of the B.Sc. Degree programme in Mathematics is a record of original project work done by us during the period of our study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda.

Gokul Venu (CCAUSMT058)

Place: Irinjalakuda

Date: 18-04-2023

ACKNOWLEDGMENT

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Gokul Venu (CCAUSMT058)

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IRREDUCIBLE POLYNOMIAL

Project report submitted to Christ College (Autonomous) in partial fulfillment of the requirement for the award of the B.Sc. Degree Programme in Mathematics

By

Lakshmi M (CCAUSMT059)

Department of Mathematics (Unaided)

Christ College (Autonomous)

Irinjalakuda

2023

CERTIFICATE

This is to certify that the entitled "IRREDUCIBLE POLYNOMIAL" Submitted to the Department of Mathematics (Unaided) in partial fulfillment of the requirement for the award of the B.Sc. Degree programme in Mathematics is a bonafide record of original research work done by Lakshmi M (CCAUSMT059) during the period of her study in the Department of Mathematics (Unaided), Christ College (Autonomous), Irinjalakuda under my supervision and guidance during the year 2022 – 2023.

Mr. Jomesh Jose	Dr. Joju K T
Adhoc Lecture	Coordinator
Department Of Mathematics (Unaided)	Department of Mathematics (Unaided)
Christ College (Autonomous),	Christ College (Autonomous),
Irinjalakuda	Irinjalakuda

External Examiner

Place: Irinjalakuda

Date: 18/04/2023

DECLARATION

I hereby declare that the project work entitled "IRREDUCIBLE POLYNOMIAL" submitted to Christ College (Autonomous) in partial fulfillment of the requirement for the award of the B.Sc. Degree programme in Mathematics is a record of original project work done by me during the period of our study in the Department of Mathematics (Unaided), Christ College (Autonomous), Irinjalakuda.

Lakshmi M (CCAUSMT059)

Place: Irinjalakuda

Date: 18/04/2023

ACKNOWLEDGEMENT

I wish to acknowledge my indebtedness to a number of colleagues who made valuable efforts and suggestions for the content of this project. There are many individuals who have come together to make this project a reality. I greatly appreciate the inspiration, support and guidance of all those people who have been instrumental in making this project a success.

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Lakshmi M (CCAUSMT059)

CONTENTS

INTRODUCTION

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This project covers the basics of rings, fields, integral domain, what irreducible polynomials are, and the various methods to find if a polynomial is irreducible or not which are, Eisenstein's criterion, the p test, and cyclotomic polynomials.

OUTLINE OF THE PROJECT

The topic we are considering for this project is Irreducible Polynomials, which is a fundamental concept in algebra and number theory.

Chapter 1 covers the basics of ring theory and gives an outlook of the field and integral domain. Since a polynomial is defined on either of these it is necessary to know about these concepts.

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GRAPH COLOURING

Project report submitted to Christ College (Autonomous) in partial fulfilment of requirement for the award of the B.Sc. Degree Programme in Mathematics by

Manikandan M (CCAUSMT060)

Department of Mathematics (Unaided)

Christ College (Autonomous)

Irinjalakuda

2023

CERTIFICATE

This is to certify that the project entitled "GRAPH COLOURING" submitted to the Department of Mathematics Unaided in partial fulfilment of the requirement for the award of the B.Sc. Degree programme in Mathematics, is a bonafide record of original research work done by Manikandan M (CCAUSMT060) during the period of their study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda under my supervision and guidance during the year 2022-2023.

Miss Mary Pauly K	Dr Joju K T
Adhoc Lecturer	Coordinator
Department of Mathematics (Unaided)	Department of Mathematics (Unaided)
Christ College (Autonomous)	Christ College(Autonomous)
Irinjalakuda	Irinjalakuda

External Examiner:

Place: Irinjalakuda

Date: 18/04/2023

DECLARATION

We hereby declare that the project work entitled "GRAPH COLOURING" submitted to the Christ College (Autonomous) in partial fulfilment of the requirement for the award of the B.Sc. Degree programme in Mathematics is a record of original project work done by us during the period of our study in the Department of Mathematics Self, Christ College (Autonomous), Irinjalakuda.

Manikandan M (CCAUSMT060)

Place: Irinjalakuda

Date: 18/04/2023

ACKNOWLEDGEMENT

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Manikandan M

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JORDAN CANONICAL FORM

Project report submitted to Christ College (Autonomous) in partial fulfilment of requirement for the award of the B.Sc. Degree Programme in Mathematics

By

Vrindha P P(CCAUSMT061)

Department of Mathematics Unaided

Christ College (Autonomous)

Irinjalakuda

2023

CERTIFICATE

This is to certify that the project entitled “JORDAN CANONICAL FORM” submitted to the Department of Mathematics Unaided in partial fulfilment of the requirement for the award of the B.Sc. Degree programme in Mathematics, is a bonafide record of original research work done by Vrindha P P (CCAUSMT061) during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda under my supervision and guidance during the year 2022-2023.

External Examiner:

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Dr. JOJU K T
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Department of Mathematics (Unaided)
Christ College (Autonomous)Irinjalakuda

Place: Irinjalakuda

Date:18-04-2023

DECLARATION

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Vrindha P P (CCAUSMT061)

Place: Irinjalakuda

Date: 18/04/2023

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