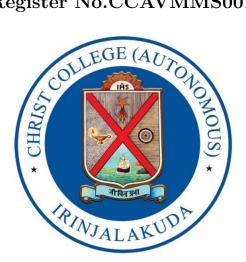
DOMINATION IN GRAPHS

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

ADWIN OUSAN

Register No.CCAVMMS001



Department of Mathematics

Christ College (Autonomous)

Irinjalakuda 2023

CERTIFICATE

This is to certify that the project entitled "DOMINATION IN GRAPHS" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of the work done by Mr. ADWIN OUSAN (CCAVMMS001) during the period of his study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023.

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External Examiner :

Place : Irinjalakuda Date : 02 june 2023

DECLARATION

I hereby declare that the project work entitled "**DOMINATION IN GRAPHS**" submitted to Christ College(Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of the work done by me during the period of my study in the Department of Mathematics, Christ College(Autonomous), Irinjalakuda.

Place : Irinjalakuda Date : 02 June 2023 ADWIN OUSAN

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First, there are no words to adequately acknowledge the wonderful grace that my Redeemer has given me. There are many individuals who have come together to make this project a reality. I greatly appreciate the inspiration; support and guidance of all those people who have been instrumental for making this project a success.

I express my deepest thanks to my guide Fr. Dr. Vincent N S, Assistant Professor, Department of Mathematics, Christ College(Autonomous), Irinjalakuda, who guided me faithfully through this entire project. I have learned so much from him, both in the subject and otherwise. Without his advice, support and guidance, it find difficult to complete this work.

I take this opportunity to express my thanks to our beloved principal Fr. Dr. Jolly Andrews CMI, who gave me the golden opportunity to do this wonderful project on the topic "**DOMINATION IN GRAPHS**"

I mark my word of gratitude to Prof. Tintumol Sunny, Head of the Department and all other teachers of the department for providing me the necessary facilities to complete this project on time.

I want to especially thank all the faculty of the library for providing various

facilities for this project.

Words cannot express the love and support I have received from my parents, whose encouragement has buoyed me up from the beginning till the end of this work.

ADWIN OUSAN

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Introduction

The theory of graph is one of the new field of mathematics. While the history of mathematics is long and sound history of mathematical graph theory has been originated by 1736 with the work of Euler's solution of konisberg bridge problem.

Any mathematical object having points and connections between them may be called graphs. Graphs are severe as mathematical modules to analyze successfully many concrete problems. In mathematics,graph theory is a study of graphs,which are mathematical structures used to model pairwise relation between objects.

In 1850, chess freaks in Europe give thought to issue for finding the least numeral of queens that is set on a chess board with a goal that each one of the blocks are either charged by a queen or inhibited by a queen. The "five queens" problem can be said to be the origin of the study of the dominating sets in graphs. Also the dominating queen problems can be stated in general as the domination of vertices of a graph.

In **Chapter 1** covers some necessary definitions, terms and concepts in domination in graph. Also we will introduces the domination number on a graph and bounds of domination number. It also covers types of domination and varieties

PRODUCT GRAPH

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc. Degree programme in Mathematics

by

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Irinjalakuda

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CERTIFICATE

This is to certify that the project entitled "**PRODUCT GRAPH**" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc. Degree programme in Mathematics, is a bonafide record of work done by **Mrs. HANNA MUSTHAFA (CCAVMMS002)** during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2021-2023

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DECLARATION

I hereby declare that the project work entitled "**PRODUCT GRAPH**" submitted to Christ College (Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of work done by me during the period of my study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda.

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I express my deepest thanks to my guide Mr. Naveen V V, Assistant Professor (Ad-hoc) Department of Mathematics, Christ College (Autonomous), Irinjalakuda, who guided me faithfully through this entire project. I have learned so much from him, both in the subject and otherwise. Without his advice, support and guidance, it find difficult to complete this work.

I mark my word of gratitude to Prof. Tintumol Sunny, Head of the Department and also Dr. Seena V, Assistant Professor, who deserves a special word of thanks for her invaluable and generous help in preparing this project in LAT_EX and all other teachers of the department for providing me the necessary facilities to complete this project on time.

I want to especially thank all the faculty of the library for providing various facilities for this project.

Words cannot express the love and support I have received from my parents, whose encouragement has buoyed me up from the beginning till the end of this work.

Hanna Musthafa

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Introduction

Graph products are applied in many areas like human genetics, a dynamic location problem etc. In this project, we consider different types of graph products and some properties of them.

Chapter 1 is a brief introduction to graph theory and Product Graph. Our main objects of study appear in chapter 2, where we introduce three fundamental graph products, namely the certain products, the direct product and the strong product. This is followed by the classification of certain associative products, providing an explanation of why the three products mentioned above are the most natural off all products. The classification also leads to a fourth product worthy of special attention, the lexicographic products. Last chapter investigates the semi ring and metric structure of the four standard products. We can have a glimpse through this product graphs.

For definitions, terminologies, notations and results, we follow mainly [1], [2], [3], [4].

MATROIDS

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

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Irinjalakuda 2023

CERTIFICATE

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Place : Irinjalakuda

Date : 2 June 2023

DECLARATION

I hereby declare that the project work entitled "MATROIDS" submitted to the Christ College(Autonomous), Irinjalakuda in partial fulfillment of the requirement for the award of Master Degree of Science in Mathematics is a record of the work done by me during the period of my study in the Department of Mathematics, Christ College(Autonomous), Irinjalakuda.

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Dr. Seena V, deserves a special word of thanks for her invaluable and generous help in preparing this project in LAT_EX .

I want to especially thank all the faculty of the library for providing various

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HRIDYA HARIPRAKASH

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Introduction

Many results of graph theory extend or simplify the theory of Matroids. These include the fundamental concepts and their examples, the strong duality between maximum matching and minimum vertex over in bipartite graphs, and the geometric duality relating planar graphs and their duals, sub matroids, and transversal matroids. Matroids arise in many contexts but are special enough to have rich combinatorial structure when a result from graph theory generalizes to matroids which can then be interpreted in other special cases. Several difficult theorems about graphs have found easier proofs using matroids.

Matroids were introduced by Whitney (1935) to study planarity and algebraic aspects of graphs by MacLane (1936) to study geometric lattices and by Vander Waerder (1937) to study independence in vector space. Most of the language comes from this context. As the word suggests Whitney conceived a matroid as an abstract generalization of a matrix, and much of the language of the theory is based on that of linear algebra. However, Whitney's approach was also to some extent motivated by his earlier work in graph theory and as a result, some of the matroid terminologies have a distinct graphical flavor. Two important papers by Rado on the combinatorial applications of matroids and finite matroids, the

Introduction

subject lay virtually dormant until Tutte, published his fundamental papers on matroids and graphs and Rado studied the representability problem for matroids. Since then interest in matroids and their applications in combinatorial theory has accelerated rapidly. This is probably due to the discovery independently by Edmonds and Fulkerson and Mirsky and Perfect of a new, important class of matroids called transversal matroids. In graph theory, the main benefit of a matroid treatment seems to be a much more natural understanding of dual concepts such as the structure of the set of cocycles or the effect of contraction of a set of edges of a graphs.

Preliminaries

- The concept of a graph and a directed or oriented graph which we call a digraph.
- We denote a graph G by a pair (V(G), E(G)) where V = V(G) is the vertex set and E = E(G) is the set of edges. The edge e = (u, v) is said to join the vertices u and v are adjacent vertices while u and v are called the endpoints of the edge e = (u, v).
- If 2 edges e_1, e_2 have a common endpoint they are said to be incident. We often denote the edge e = (u, v) by uv or vu.
- A loop of a graph is an edge of the type (x, x). Two edges are parallel if they have common endpoints and are not loops.
- A graph is simple if it has no loops or parallel edges.
- The degree of vertex v is the number of edges having G as an endpoint, and is denoted by deg(v).
- A graph is regular if all its vertices have the same degree.
- Two graph G_1, G_2 are isomorphic if there is a bijection $\phi: V(G_1) \to V(G_2)$

Nets And Filters

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

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This is to certify that the project entitled "Nets And Filters" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of work done by Ms.KAVITHA E H (CCAVMMS004) during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023

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DECLARATION

I hereby declare that the project work entitled "**Nets And Filters**" submitted to Christ College(Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of work done by me during the period of my study in the Department of Mathematics, Christ College(Autonomous), Irinjalakuda.

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First, there are no words to adequately acknowledge the wonderful grace that my Redeemer has given me. There are many individuals who have come together to make this project a reality. I greatly appreciate the inspiration; support and guidance of all those people who have been instrumental for making this project a success.

I express my deepest thanks to my guide Dr. Seena V, Assistant Professor, Department of Mathematics, Christ College(Autonomous), Irinjalakuda, who guided me faithfully through this entire project. I have learned so much from her, both in the subject and otherwise. Without her advice, support and guidance, it find difficult to complete this work.

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My project guide Dr. Seena V, deserves a special word of thanks for her invaluable and generous help in preparing this project in LAT_EX .

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Kavitha E H

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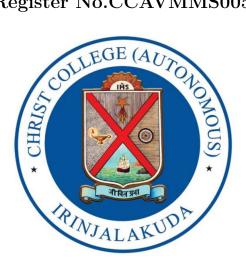
FUZZY LOGIC

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

NOBLE JOSE

Register No.CCAVMMS005



Department of Mathematics

Christ College (Autonomous)

Irinjalakuda 2023

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This is to certify that the project entitled "Fuzzy Logic" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of original work done by Mr.NOBLE JOSE (CCAVMMS005) during the period of his study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023

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I hereby declare that the project work entitled "**Fuzzy Logic**" submitted to Christ College(Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of work done by me during the period of my study in the Department of Mathematics, Christ College(Autonomous), Irinjalakuda.

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First, there are no words to adequately acknowledge the wonderful grace that my Redeemer has given me. There are many individuals who have come together to make this project a reality. I greatly appreciate the inspiration; support and guidance of all those people who have been instrumental for making this project a success.

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I want to especially thank all the faculty of the library for providing various

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NOBLE JOSE

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GRAPH COLOURING

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

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2023

CERTIFICATE

This is to certify that the project entitled "GRAPH COLOURING" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of the work done by Mr.SANJAY SOJAN (CCAVMMS006) during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023.

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Place : Irinjalakuda

Date : 2 June 2023

DECLARATION

I hereby declare that the project work entitled "GRAPH COLOURING" submitted to Christ College(Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of the work done by me during the period of my study in the Department of Mathematics, Christ College(Autonomous), Irinjalakuda.

Place : Irinjalakuda Date : 2 June 2023 Sanjay Sojan

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I take this opportunity to express my thanks to our beloved principal Fr. Dr. Jolly Andrews CMI, who gave me the golden opportunity to do this wonderful project on the topic "GRAPH COLOURING"

I mark my word of gratitude to Prof. Tintumol Sunny, Head of the Department and all other teachers of the department for providing me the necessary facilities to complete this project on time.

Dr. Seena V, Assistant Professor deserves a special word of thanks for her invaluable and generous help in preparing this project in LAT_EX .

I want to especially thank all the faculty of the library for providing various

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Words cannot express the love and support I have received from my parents, whose encouragement has buoyed me up from the beginning till the end of this work.

Sanjay Sojan

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Introduction

The subject graph theory emerged while solving a problem associated with a bridge in konisberg, which was situated in Russia. In the year 1735 Euler considered this problem and constructed a structure to solve this problem. As a result the first formal 'graph' structure had been drawn, and a new branch of mathematics started its journey. Over the years, the theory of graphs have a tremendous growth in various directions. The structures-the graphs possesing special properties got attention of graph theorists as they found these graphs are very useful in studing many concepts in social and scientific scenario. The complete graphs and bipartite graphs introduced by A.F. Mobius have more recreational problems. Trees are useful in the calculation of currents in electrical works. Cayley studied particular analytical forms from differential calculus to study the trees. Based on the characterizations and applications of graphs, new areas of graph theory such as extremal graph theory, enumerative graph theory and random graph theory have been developed.

The graph colouring introduced by Arther Cayley is the process of assigning vertices of a graph such that no adjacent vertices have the same colour. The goal is to use the smallest number of colours possible to colour the graph, which is

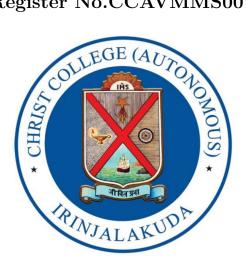
UMBRAL CALCULUS

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc. Degree programme in Mathematics

by

SEYIN.C.J.

Register No.CCAVMMS007



Department of Mathematics

Christ College (Autonomous)

Irinjalakuda 2023

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This is to certify that the project entitled "UMBRAL CALCULUS" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc. Degree programme in Mathematics, is a bonafide record of work done by Ms. SEYIN.C.J. (CCAVMMS007) during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023.

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I hereby declare that the project work entitled "UMBRAL CALCULUS" submitted to Christ College (Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of work done by me during the period of my study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda.

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Dr. Seena V, deserves a special word of thanks for her invaluable and generous help in preparing this project in LATEX.

I want to especially thank all the faculty of the library for providing various

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Words cannot express the love and support I have received from my parents, whose encouragement has buoyed me up from the beginning till the end of this work.

SEYIN.C.J.

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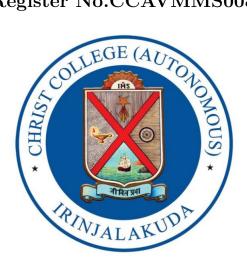
Variational Optimization

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

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Sherin P. Johnson

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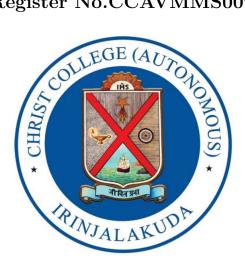
Algebraic Number Theory

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

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I express my deepest thanks to my guide, Prof. Tintumol Sunny, Head of the Department, Department of Mathematics, Christ College(Autonomous), Irinjalakuda, who guided me faithfully through this entire project. I have learned so much from her, both in the subject and otherwise. Without her advice, support and guidance, it find difficult to complete this work.

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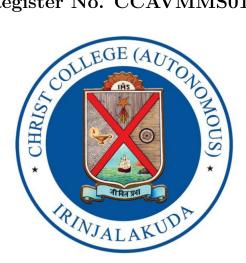
CAYLEY GRAPH

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

SREELAKSHMI K P

Register No. CCAVMMS010



Department of Mathematics

Christ College (Autonomous)

Irinjalakuda 2023

CERTIFICATE

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DECLARATION

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Place : Irinjalakuda Date : 02 June 2023 SREELAKSHMI K P

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Words cannot express the love and support I have received from my parents, whose encouragement has buoyed me up from the beginning till the end of this work.

SREELAKSHMI K P

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Introduction

Graph theory has become a very popular and rapidly growing area of Discrete Mathematics for its numerous theoretical development and countless application to practical problems. As a research area, graph theory is still relatively young, but it is maturing rapidly with many deep results have been discovered over the last couple of decades. The study of graph theory was introduced by Euler in 1736.For the contribution of Euler towards graph theory , he is known as the father of graph theory. But the term graph was introduced by Sylvester in paper published in 1878.It took 200 years since Euler's published in 1936 by Dencs konig.

The definition of Cayley graph was introduced by Arther Cayley in 1878 to explain the concept of abstract groups which described by a set of generators .In the last 50 years , the theory of cayley graphs has been grown in to substantial branch of algebraic graph theory .

A group is a set of elements together with an operation that combines any two of its elements to form a third element. The set and operation must satisfy group axioms, namely associativity, identity and inverse elements. Cayley graph depicts the elements of the groups as vertexes connected by colored edges. Every

PHYLOGENETICS

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

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SREELAKSHMI VALSAN

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Introduction

Graphs come in two flavors, undirected and directed. One type of undirected graph that plays an important role is unrooted trees. In the context of directed graphs, we discuss the concept of a directed, acyclic graph (DAG) and then introduce rooted trees as an important example. We introduce a number of different kinds of traversals of trees or DAGs that are often used in algorithms. This chapter concludes by introducing the concepts of taxa, clusters, clades and splits.

Phylogenetic analysis aims at uncovering the evolutionary relationships between different species or taxa, to obtain an understanding of the evolution of life on Earth. Phylogenetic trees are widely used to address this task and are usually computed from molecular sequences. They also have applications in many other areas. For example, they are used to determine the age and rate of diversification of taxa, to understand the evolutionary history of gene families, in sequence-analysis methods to allow phylogenetic footprinting, in epidemiology to trace the origin and transmission of infectious diseases, or to study the coevolution of hosts and parasites. Introduction

In the literature, the term phylogenetic network is defined and used in a number of different ways, usually focusing on the specific type of network For example, a phylogenetic network is sometimes too narrowly defined as a rooted DAG whose leaves are labeled by taxa. The concept of a split plays an important role in the mathematics of phylogeny. It is motivated by the simple, but crucial, observation that every edge e in an unrooted phylogenetic tree T defines a bipartition of the underlying taxon set \mathcal{X} into two non-empty and disjoint subsets, A and B, known as a split. The splits of an unrooted phylogenetic tree uniquely define the topology of the tree and splits are used, for example, to compare different trees or to compute consensus trees. Any set of splits that is compatible corresponds to a phylogenetic tree and so one possible way to generalize from trees to networks is to consider sets of splits that are incompatible.

We present two different approaches. The first is the convex hull algorithm that computes the Buneman graph and can be applied to any set of splits, using an exponential number of nodes and edges in the worst case It is also used to compute median networks. The second is the circular network algorithm, which can be applied to any set of circular splits and produces an outer-labeled planar network with only a quadratic number of nodes and edges.

A typical application of a cluster network is that one is given multiple rooted gene trees for a set of species and one would like to produce a network to illustrate the parts of the phylogeny upon which the trees agree and which parts are resolved in different ways.

 $\mathbf{2}$

Algebraic Topology

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

AMEGHA K

Register No.CCAVMMS012



Department of Mathematics

Christ College (Autonomous)

Irinjalakuda

2023

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This is to certify that the project entitled "Algebraic Topology" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of work done by Ms. AMEGHA K (CCAVMMS012) during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023

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DECLARATION

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Place : Irinjalakuda Date : 02 June 2023 Amegha K

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I express my deepest thanks to my guide, Fr. Dr. Vincent N S, Assistant Professor, Department of Mathematics, Christ College(Autonomous), Irinjalakuda, who guided me faithfully through this entire project. I have learned so much from him, both in the subject and otherwise. Without his advice, support and guidance, it find difficult to complete this work.

I take this opportunity to express my thanks to our beloved principal Fr. Dr. Jolly Andrews CMI, who gave me the golden opportunity to do this wonderful project on the topic "Algebraic Topology"

I mark my word of gratitude to Prof. Tintumol Sunny, Head of the Department and all other teachers of the department for providing me the necessary facilities to complete this project on time.

Dr. Seena V, Assistant professor, deserves a special word of thanks for his invaluable and generous help in preparing this project in LAT_EX .

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Amegha K

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Eigenvalues of Graphs

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc. Degree programme in Mathematics

by

ANJANA BENNY

Register No.CCAVMMS013



Department of Mathematics

Christ College (Autonomous)

Irinjalakuda

2023

CERTIFICATE

This is to certify that the project entitled "Eigenvalues of Graphs" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc. Degree programme in Mathematics, is a bonafide record of work done by Ms.ANJANA BENNY (CCAVMMS013) during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023.

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Place : Irinjalakuda Date : 2 June 2023

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I hereby declare that the project work entitled "**Eigenvalues of Graphs**" submitted to Christ College (Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of work done by me during the period of my study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda.

Place : Irinjalakuda Date : 2 June 2023 Anjana Benny

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First, there are no words to adequately acknowledge the wonderful grace that my Redeemer has given me. There are many individuals who have come together to make this project a reality. I greatly appreciate the inspiration; support and guidance of all those people who have been instrumental for making this project a success.

I express my deepest thanks to my guide Dr. Seena V, Assistant professor, Department of Mathematics, Christ College (Autonomous), Irinjalakuda, who guided me faithfully through this entire project. I have learned so much from her, both in the subject and otherwise. Without her advice, support, and guidance, it find difficult to complete this work.

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I mark my word of gratitude to Prof. Tintumol Sunny, Head of the Department, and all other teachers of the department for providing me the necessary facilities to complete this project on time.

My project guide, Dr. Seena V, deserves a special word of thanks for her invaluable and generous help in preparing this project in LATEX.

I want to especially thank all the faculty of the library for providing various

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Anjana Benny

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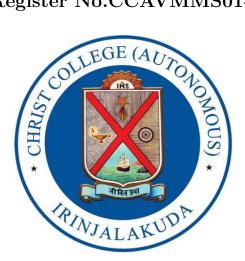
Bilinear forms

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

ANN ROSE SHAJU PANADAN

Register No.CCAVMMS014



Department of Mathematics

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Irinjalakuda 2023

CERTIFICATE

This is to certify that the project entitled "BILINEAR FORMS" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of work done by Ms.ANN ROSE SHAJU PANADAN (CCAVMMS014) during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023.

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DECLARATION

I hereby declare that the project work entitled "**BILINEAR FORMS**" submitted to Christ College(Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of original project work done by me during the period of my study in the Department of Mathematics, Christ College(Autonomous), Irinjalakuda.

Place : Irinjalakuda Date 2 June 2023 ANN ROSE SHAJU PANADAN

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First, there are no words to adequately acknowledge the wonderful grace that my Redeemer has given me. There are many individuals who have come together to make this project a reality. I greatly appreciate the inspiration; support and guidance of all those people who have been instrumental for making this project a success.

I express my deepest thanks to my guide Prof.Tintumol Sunny, Head of Department, Department of Mathematics, Christ College(Autonomous), Irinjalakuda, who guided me faithfully through this entire project. I have learned so much from her, both in the subject and otherwise. Without her advice, support and guidance, it find difficult to complete this work.

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ANN ROSE SHAJU PANADAN

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Fundamental Groups and Covering Spaces

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc. Degree programme in Mathematics

by

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Register No.CCAVMMS015



Department of Mathematics

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Irinjalakuda

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This is to certify that the project entitled "Fundamental Groups and Covering Spaces" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc. Degree programme in Mathematics, is a bonafide record of project work done by Ms. ANUPAMA ROY (CCAVMMS015) during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2021-2023.

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I hereby declare that the project work entitled "Fundamental Groups and Covering Spaces" submitted to Christ College (Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of project work done by me during the period of my study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda.

Place : Irinjalakuda Date : 02 June 2023 Anupama Roy

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Anupama Roy

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Introduction to lie algebra

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

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Register No.CCAVMMS016



Department of Mathematics

Christ College (Autonomous)

Irinjalakuda

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This is to certify that the project entitled "Introduction to Lie Algebra" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of work done by Ms.ANUSREE A D (CCAVMMS016) during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023

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DECLARATION

I hereby declare that the project work entitled "Introduction to Lie Algebra" submitted to Christ College(Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of work done by me during the period of my study in the Department of Mathematics, Christ College(Autonomous), Irinjalakuda.

Place : Irinjalakuda Date : 02 June 2023 ANUSREE A D

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First, there are no words to adequately acknowledge the wonderful grace that my Redeemer has given me. There are many individuals who have come together to make this project a reality. I greatly appreciate the inspiration; support and guidance of all those people who have been instrumental for making this project a success.

I express my deepest thanks to my guide Dr.Shinto K G, Assistant Professor, Department of Mathematics, Christ College(Autonomous), Irinjalakuda, who guided me faithfully through this entire project. I have learned so much from him, both in the subject and otherwise. Without his advice, support and guidance, it find difficult to complete this work.

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Anusree A D

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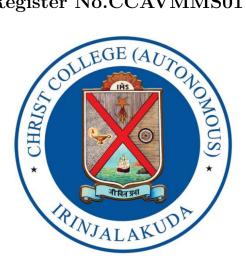
Fixed Point Theorem

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

ASWIN V.G

Register No.CCAVMMS017



Department of Mathematics

Christ College (Autonomous)

Irinjalakuda 2023

CERTIFICATE

This is to certify that the project entitled "Fixed Point Theorem" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of work done by Mr.ASWIN V .G (CCAVMMS017) during the period of his study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023

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DECLARATION

I hereby declare that the project work entitled "**Fixed Point Theorem**" submitted to Christ College(Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of work done by me during the period of my study in the Department of Mathematics, Christ College(Autonomous), Irinjalakuda.

Place : Irinjalakuda Date : 02 June 2023 Aswin V .G

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Aswin V .G

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CODING THEORY

Project report submitted to Christ College (Autonomous) in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

CHRISTY DAVI

Register No. CCAVMMS018



Department of Mathematics

Christ College (Autonomous)

Irinjalakuda 2023

CERTIFICATE

This is to certify that the project entitled "CODING THEORY" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of the work done by Ms. CHRISTY DAVI (CCAVMMS018) during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023.

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DECLARATION

I hereby declare that the project work entitled "CODING THEORY" submitted to Christ College(Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of the work done by me during the period of my study in the Department of Mathematics, Christ College(Autonomous), Irinjalakuda.

Place : Irinjalakuda Date : 02 June 2023 CHRISTY DAVI

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Dr. Seena V, Assistant Professor, deserves a special word of thanks for her invaluable and generous help in preparing this project in LATEX.

I want to especially thank all the faculty of the library for providing various

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CHRISTY DAVI

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Digital Signatures

Project report submitted to Christ College (Autonomous) in partial fulfillment of the requirement for the award of the M.Sc Degree programme in Mathematics

by

EVELYN JOSE

Register No.CCAVMMS019



Department of Mathematics

Christ College (Autonomous)

Irinjalakuda

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CERTIFICATE

This is to certify that the project entitled "**Digital Signatures**" submitted to the Department of Mathematics in partial fulfillment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of work done by **Ms. EVELYN JOSE (CCAVMMS019)** during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023.

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DECLARATION

I hereby declare that the project work entitled "**Digital Signatures**" submitted to Christ College(Autonomous), Irinjalakuda in partial fulfillment of the requirement for the award of Master Degree of Science in Mathematics is a record of work done by me during the period of my study in the Department of Mathematics, Christ College(Autonomous), Irinjalakuda.

Place : Irinjalakuda Date : 2 June 2023 Evelyn Jose

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Evelyn Jose

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