

DOMINATION IN GRAPHS

Project report submitted to Christ College (Autonomous) in partial
fulfilment of the requirement for the award of the M.Sc Degree
programme in Mathematics

by

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2023

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This is to certify that the project entitled “**DOMINATION IN GRAPHS**” submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of the work done by **Mr. ADWIN OUSAN (CCAVMMS001)** during the period of his study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023.

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I express my deepest thanks to my guide Fr. Dr. Vincent N S, Assistant Professor, Department of Mathematics, Christ College(Autonomous), Irinjalakuda, who guided me faithfully through this entire project. I have learned so much from him, both in the subject and otherwise. Without his advice, support and guidance, it find difficult to complete this work.

I take this opportunity to express my thanks to our beloved principal Fr. Dr. Jolly Andrews CMI, who gave me the golden opportunity to do this wonderful project on the topic “**DOMINATION IN GRAPHS**”

I mark my word of gratitude to Prof. Tintumol Sunny, Head of the Department and all other teachers of the department for providing me the necessary facilities to complete this project on time.

Dr.Seena V, Assistant Professor, deserves a special word of thanks for his invaluable and generous help in preparing this project in \LaTeX .

I want to especially thank all the faculty of the library for providing various

facilities for this project.

Words cannot express the love and support I have received from my parents, whose encouragement has buoyed me up from the beginning till the end of this work.

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Introduction

The theory of graph is one of the new field of mathematics. While the history of mathematics is long and sound history of mathematical graph theory has been originated by 1736 with the work of Euler's solution of konisberg bridge problem.

Any mathematical object having points and connections between them may be called graphs. Graphs are severe as mathematical modules to analyze successfully many concrete problems. In mathematics,graph theory is a study of graphs,which are mathematical structures used to model pairwise relation between objects.

In 1850, chess freaks in Europe give thought to issue for finding the least numeral of queens that is set on a chess board with a goal that each one of the blocks are either charged by a queen or inhibited by a queen. The "five queens" problem can be said to be the origin of the study of the dominating sets in graphs.Also the dominating queen problems can be stated in general as the domination of vertices of a graph.

In **Chapter 1** covers some necessary definitions,terms and concepts in domination in graph. Also we will introduces the domination number on a graph and bounds of domination number. It also covers types of domination and varieties

PRODUCT GRAPH

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Hanna Musthafa

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Introduction

Graph products are applied in many areas like human genetics, a dynamic location problem etc. In this project, we consider different types of graph products and some properties of them.

Chapter 1 is a brief introduction to graph theory and Product Graph. Our main objects of study appear in chapter 2, where we introduce three fundamental graph products, namely the certain products, the direct product and the strong product. This is followed by the classification of certain associative products, providing an explanation of why the three products mentioned above are the most natural off all products. The classification also leads to a fourth product worthy of special attention, the lexicographic products. Last chapter investigates the semi ring and metric structure of the four standard products. We can have a glimpse through this product graphs.

For definitions, terminologies, notations and results, we follow mainly [1], [2], [3], [4].

MATROIDS

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Introduction

Many results of graph theory extend or simplify the theory of Matroids. These include the fundamental concepts and their examples, the strong duality between maximum matching and minimum vertex cover in bipartite graphs, and the geometric duality relating planar graphs and their duals, sub matroids, and transversal matroids. Matroids arise in many contexts but are special enough to have rich combinatorial structure when a result from graph theory generalizes to matroids which can then be interpreted in other special cases. Several difficult theorems about graphs have found easier proofs using matroids.

Matroids were introduced by Whitney (1935) to study planarity and algebraic aspects of graphs by MacLane (1936) to study geometric lattices and by Vander Waerder (1937) to study independence in vector space. Most of the language comes from this context. As the word suggests Whitney conceived a matroid as an abstract generalization of a matrix, and much of the language of the theory is based on that of linear algebra. However, Whitney's approach was also to some extent motivated by his earlier work in graph theory and as a result, some of the matroid terminologies have a distinct graphical flavor. Two important papers by Rado on the combinatorial applications of matroids and finite matroids, the

subject lay virtually dormant until Tutte, published his fundamental papers on matroids and graphs and Rado studied the representability problem for matroids. Since then interest in matroids and their applications in combinatorial theory has accelerated rapidly. This is probably due to the discovery independently by Edmonds and Fulkerson and Mirsky and Perfect of a new, important class of matroids called transversal matroids. In graph theory, the main benefit of a matroid treatment seems to be a much more natural understanding of dual concepts such as the structure of the set of cocycles or the effect of contraction of a set of edges of a graphs.

Preliminaries

- The concept of a graph and a directed or oriented graph which we call a digraph.
- We denote a graph G by a pair $(V(G), E(G))$ where $V = V(G)$ is the vertex set and $E = E(G)$ is the set of edges. The edge $e = (u, v)$ is said to join the vertices u and v are adjacent vertices while u and v are called the endpoints of the edge $e = (u, v)$.
- If 2 edges e_1, e_2 have a common endpoint they are said to be incident. We often denote the edge $e = (u, v)$ by uv or vu .
- A loop of a graph is an edge of the type (x, x) . Two edges are parallel if they have common endpoints and are not loops.
- A graph is simple if it has no loops or parallel edges.
- The degree of vertex v is the number of edges having G as an endpoint, and is denoted by $deg(v)$.
- A graph is regular if all its vertices have the same degree.
- Two graph G_1, G_2 are isomorphic if there is a bijection $\phi : V(G_1) \rightarrow V(G_2)$

Nets And Filters

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Kavitha E H

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FUZZY LOGIC

Project report submitted to Christ College (Autonomous) in partial
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GRAPH COLOURING

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Dr. Seena V, Assistant Professor deserves a special word of thanks for her invaluable and generous help in preparing this project in *L_AT_EX*.

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Introduction

The subject graph theory emerged while solving a problem associated with a bridge in Konigsberg, which was situated in Russia. In the year 1735 Euler considered this problem and constructed a structure to solve this problem. As a result the first formal 'graph' structure had been drawn, and a new branch of mathematics started its journey. Over the years, the theory of graphs have a tremendous growth in various directions. The structures-the graphs possessing special properties got attention of graph theorists as they found these graphs are very useful in studying many concepts in social and scientific scenario. The complete graphs and bipartite graphs introduced by A.F. Mobius have more recreational problems. Trees are useful in the calculation of currents in electrical works. Cayley studied particular analytical forms from differential calculus to study the trees. Based on the characterizations and applications of graphs, new areas of graph theory such as extremal graph theory, enumerative graph theory and random graph theory have been developed.

The graph colouring introduced by Arthur Cayley is the process of assigning vertices of a graph such that no adjacent vertices have the same colour. The goal is to use the smallest number of colours possible to colour the graph, which is

UMBRAI CALCULUS

Project report submitted to Christ College (Autonomous) in partial
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Variational Optimization

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Algebraic Number Theory

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CAYLEY GRAPH

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Introduction

Graph theory has become a very popular and rapidly growing area of Discrete Mathematics for its numerous theoretical development and countless application to practical problems. As a research area, graph theory is still relatively young, but it is maturing rapidly with many deep results have been discovered over the last couple of decades. The study of graph theory was introduced by Euler in 1736. For the contribution of Euler towards graph theory, he is known as the father of graph theory. But the term graph was introduced by Sylvester in paper published in 1878. It took 200 years since Euler's published in 1936 by Dencs konig.

The definition of Cayley graph was introduced by Arther Cayley in 1878 to explain the concept of abstract groups which described by a set of generators. In the last 50 years, the theory of cayley graphs has been grown in to substantial branch of algebraic graph theory.

A group is a set of elements together with an operation that combines any two of its elements to form a third element. The set and operation must satisfy group axioms, namely associativity, identity and inverse elements. Cayley graph depicts the elements of the groups as vertexes connected by colored edges. Every

PHYLOGENETICS

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Introduction

Graphs come in two flavors, undirected and directed. One type of undirected graph that plays an important role is unrooted trees. In the context of directed graphs, we discuss the concept of a directed, acyclic graph (DAG) and then introduce rooted trees as an important example. We introduce a number of different kinds of traversals of trees or DAGs that are often used in algorithms. This chapter concludes by introducing the concepts of taxa, clusters, clades and splits.

Phylogenetic analysis aims at uncovering the evolutionary relationships between different species or taxa, to obtain an understanding of the evolution of life on Earth. Phylogenetic trees are widely used to address this task and are usually computed from molecular sequences. They also have applications in many other areas. For example, they are used to determine the age and rate of diversification of taxa, to understand the evolutionary history of gene families, in sequence-analysis methods to allow phylogenetic footprinting, in epidemiology to trace the origin and transmission of infectious diseases, or to study the co-evolution of hosts and parasites.

In the literature, the term phylogenetic network is defined and used in a number of different ways, usually focusing on the specific type of network. For example, a phylogenetic network is sometimes too narrowly defined as a rooted DAG whose leaves are labeled by taxa. The concept of a split plays an important role in the mathematics of phylogeny. It is motivated by the simple, but crucial, observation that every edge e in an unrooted phylogenetic tree T defines a bipartition of the underlying taxon set \mathcal{X} into two non-empty and disjoint subsets, A and B , known as a split. The splits of an unrooted phylogenetic tree uniquely define the topology of the tree and splits are used, for example, to compare different trees or to compute consensus trees. Any set of splits that is compatible corresponds to a phylogenetic tree and so one possible way to generalize from trees to networks is to consider sets of splits that are incompatible.

We present two different approaches. The first is the convex hull algorithm that computes the Buneman graph and can be applied to any set of splits, using an exponential number of nodes and edges in the worst case. It is also used to compute median networks. The second is the circular network algorithm, which can be applied to any set of circular splits and produces an outer-labeled planar network with only a quadratic number of nodes and edges.

A typical application of a cluster network is that one is given multiple rooted gene trees for a set of species and one would like to produce a network to illustrate the parts of the phylogeny upon which the trees agree and which parts are resolved in different ways.

Algebraic Topology

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I express my deepest thanks to my guide, Fr. Dr. Vincent N S, Assistant Professor, Department of Mathematics, Christ College(Autonomous), Irinjalakuda, who guided me faithfully through this entire project. I have learned so much from him, both in the subject and otherwise. Without his advice, support and guidance, it find difficult to complete this work.

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Eigenvalues of Graphs

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Bilinear forms

Project report submitted to Christ College (Autonomous) in partial
fulfilment of the requirement for the award of the M.Sc Degree
programme in Mathematics

by

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Register No.CCAVMMS014



Department of Mathematics

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This is to certify that the project entitled “**BILINEAR FORMS**” submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of work done by **Ms.ANN ROSE SHAJU PANADAN (CCAVMMS014)** during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023.

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I hereby declare that the project work entitled “**BILINEAR FORMS**” submitted to Christ College(Autonomous), Irinjalakuda in partial fulfilment of the requirement for the award of Master Degree of Science in Mathematics is a record of original project work done by me during the period of my study in the Department of Mathematics, Christ College(Autonomous), Irinjalakuda.

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I express my deepest thanks to my guide Prof.Tintumol Sunny, Head of Department, Department of Mathematics, Christ College(Autonomous), Irinjalakuda, who guided me faithfully through this entire project. I have learned so much from her, both in the subject and otherwise. Without her advice, support and guidance, it find difficult to complete this work.

I take this opportunity to express my thanks to our beloved principal Fr. Dr. Jolly Andrews CMI, who gave me the golden opportunity to do this wonderful project on the topic “**Bilinear forms**”

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Fundamental Groups and Covering Spaces

Project report submitted to Christ College (Autonomous) in partial
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Introduction to lie algebra

Project report submitted to Christ College (Autonomous) in partial
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Fixed Point Theorem

Project report submitted to Christ College (Autonomous) in partial
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First, there are no words to adequately acknowledge the wonderful grace that my Redeemer has given me. There are many individuals who have come together to make this project a reality. I greatly appreciate the inspiration; support and guidance of all those people who have been instrumental for making this project a success.

I express my deepest thanks to my guide Dr.Shinto K G, Assistant Professor, Department of Mathematics, Christ College(Autonomous), Irinjalakuda, who guided me faithfully through this entire project. I have learned so much from him, both in the subject and otherwise. Without his advice, support and guidance, it find difficult to complete this work.

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Aswin V .G

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CODING THEORY

Project report submitted to Christ College (Autonomous) in partial
fulfilment of the requirement for the award of the M.Sc Degree
programme in Mathematics

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This is to certify that the project entitled “**CODING THEORY**” submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of the work done by **Ms. CHRISTY DAVI (CCAVMMS018)** during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023.

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CHRISTY DAVI

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Digital Signatures

Project report submitted to Christ College (Autonomous) in partial
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programme in Mathematics

by

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This is to certify that the project entitled “**Digital Signatures**” submitted to the Department of Mathematics in partial fulfillment of the requirement for the award of the M.Sc Degree programme in Mathematics, is a bonafide record of work done by **Ms. EVELYN JOSE (CCAVMMS019)** during the period of her study in the Department of Mathematics, Christ College (Autonomous), Irinjalakuda, under my supervision and guidance during the year 2022-2023.

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Evelyn Jose

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