PHY2C07:STATISTICAL MECHANICS

A Part

- 1. Define the term "equal a priori probability".
- 2. What is an ensemble? What is the relevance in the statistical approach?
- 3. What do you mean by Chemical potential?
- 4. What is a microcanonical ensemble?
- 5. Derive the bridging relation between entropy and number of microstates for a system.
- 6. Differentiate between microstate and macrostate with reference to an ensemble.
- 7. Explain the concept of microstate and macrostate
- 8. State and explain the postulates of equal a priori probability.
- 9. Give statistical definition of Entropy
- 10. Mention any four implications of the formula $S = k \ln \Omega$
- 11. How the physical reason for reducing the number of microstates in resolving the Gibbs paradox is explained?
- 12. How the physical reason for reducing the number of microstates in resolving the Gibbs paradox is explained?
- 13. How the Gibbs paradox is resolved?
- 14. what is mean by Gibbs Paradox
- 15. What is meant by phase space?
- 16. What do you mean by the term phase space and resentation point
- 17. Write a short note on statistical ensembles.
- 18. How the uncertainty principle plays an important role in finding the number of representative points in phase space?
- 19. Show that in a steady state, probability density is independent of co-ordinates of phase space
- 20. State and explain Liouville's theorem.
- 21. What is phase space.
- 22. What is the minimum volume required by a particle to occupy in two dimensional space
- 23. Differentiate microeconomics and Canonical ensembles.
- 24. Define micro cononical ensemble
- 25. Derive the expression for partition function of a classical ideal gas using formulation of canonical ensemble.
- 26. Define Canonical ensemble
- 27. Write down the partition function of a two level system of energies 0 and ϵ .
- 28. Derive the expression for the partition function of classical ideal gas using the formulation of Canonical ensemble.
- 29. Bring out the statistical origin of IIIrd law of thermodynamics.
- 30. Write the expression which shows the entropy of a physical system is solely and completely determined by the probability values of its accessible dynamical states. What conclusions can be derived from it?
- 31. Write an expression for partition function and explain the terms.

- 32. Show that ensemble average of any physical quantity f is identical with the value one expects to obtain on making an appropriate measurement on the system.
- 33. explain virial theorem.
- 34. Explain the virial theorem of Clausius. How can it be applied to classical ideal gas?
- 35. Using equi-partition theorem, find Cv of a monoatomic ideal gas
- 36. state and explain equipartition theorem
- 37. State Equipartition theorem.
- 38. illustrate grant Canonical ensemble with an example.
- 39. What is fugacity? How it related to the grant partition function of the system.
- 40. Define Grand Canonical ensemble
- 41. How is fugacity of a system related to q potential?
- 42. Write an expression for grand partition function and explain the terms.
- 43. what is meant by density operator
- 44. Derive the equation of motion for the density matrix
- 45. Define density matrix.
- 46. Show that $i\hbar \dot{\hat{\rho}} = [\hat{H}, \hat{\rho}]$
- 47. Define Density Operator
- 48. Explain the postulate of random phases.
- 49. What is occupation number?
- 50. Explain the statistics of occupation numbers.
- 51. Two particles are to be distributed in 3 cells. How many microstates are possible is the particles are (a)Bosons (b)Fermions (c)Boltzons
- 52. Consider a system of 3 fermions which can occupy any of the 4 available energy states with equal probability. What is the entropy of the system?
- ^{53.} Explain briefly an ideal gas in micro canonical mechanical ensemble.
- 54. Write down the anti symmetric wave function using Slater determinant.
- 55. What is the expression for q potential for Bose Einstein Fermi Dirac and Maxwell Boltzmann cases.
- 56. Consider a system of 3 fermions which can occupy any of the 4 available energy states with equal probability. What is the entropy of the system?
- 57. Explain briefly an ideal gas in other quantum-mechanical ensembles.
- 58. Explain the statistics of occupation number
- 59. Discuss the statistics of the occupation numbers
- 60. Explain Bose Einstein condensation.
- 61. Explain the onset condition for Bose Einstein condensation.
- 62. Give the condition for onset of Bose Einstein condensation. Discuss about the physical state of the system below characteristic temperature.

- 63. What is Bose Einstein condensation?
- 64. what is meant by Bose Einstein condensation
- 65. Prove that the pressure of an ideal Bose gas at the transition temperature is about one half of that of an equivalent Boltzmannian gas.
- 66. Show the variation of specific heat for an ideal Bose gas as a function of temperature.
- 67. What is Bose-Einstein condensation?
- 68. What is Stefan Boltzmann law?
- 69. Explain the Debye's model of Phonon gas
- 70. What are phonons?
- 71. State Debyes law for specific heat
- 72. Explain the Debye's modification of Einstein's model for specific heat capacity of solids
- 73. Explain the Debye's modification of Einstein's model for specific heat of solids
- 74. State Debyes law for specific heat.
- 75. Compare Debye theory of specific heat capacity of solids with that of Einstein.
- 76. Compare Einstein's and Debyes model of phonon gas
- 77. Graphically represent the variation of specific heat capacity of an ideal Fermi gas with temperature.
- 78. Obtain and expression for Fermi momentum of a system in terms of number density.
- 79. Define fermi temperature and fermi energy. How fermi energy is related to chemical potential.
- 80. 'Even at absolute zero, the Fermi system is quite live'. Justify
- 81. Obtain the equation of an ideal Fermi gas at high temperature and low density case.
- 82. Fermi systems are quite lively at absolute zero. Justify
- 83. Obtain and expression for Fermi momentum of a system in terms of number density.
- 84. Show that for a Fermi gas $\frac{PV}{NKT} = \frac{f_{5/2}(z)}{f_{3/2}(z)}$
- 85. Distinguish between weakly degenerate, degenerate and strongly degenerate Fermi gas.
- 86. Derive the expression for the Fermi energy of the system.
- 87. What is Fermi energy? Explain its physical significance
- 88. Explain Pauli diamagnetism.
- 89. Discuss the magnetic behaviour of ideal fermi gas
- 90. Obtain an expression for susceptibility of a diamagnet.
- 91. Explain Landau diamagnetism.
- 92. Write a short note on specific heat of electron gas in metals.
- 93. Give the expression for specific heat of a metallic solid.
- 94. Give the expression for specific heat for a metallic solid

95. Discuss qualitatively the basic idea behind the paramagnetic behaviour of an ideal fermi gas

B Part

- 96. Explain Gibb's paradox using the idea of entropy of mixing. How is the paradox resolved? Will there be Gibbs paradox if we use quantum statistics for ideal gas.
- 97. State and prove Liouville's theorem. Discuss any one consequence of the same.
- 98. State and prove Liouville's theorem. Discuss the consequences of the theory.
- 99. (a) Explain the concept of microcanonical ensemble. (b) State liouville's theorem and discuss its consequences.
- 100. Obtain thermodynamics of classical ideal gas considering the system as the member of microcanonical ensemble.
- 101. Compare the classical and quantum mechanical theories of paramagnetism.
- 102. Discuss the harmonic oscillator problem by both classically and quantum mechanically using canonical ensemble formulation.
- 103. Discuss the Classical and Quantum concepts of paramagnetism.
- 104. Discuss the problem of paramagnetism using the formulation of the canonical ensemble
- 105. Show that for large N, canonical ensemble is equvallent to microcanonical ensemble.
- 106. Discuss about equipartition theorem and virial theorem. Show that V=-2K for a classical ideal gas.
- 107. Discuss the fluctuations of energy in the canonical ensemble. Show that in the thermodynamic limit, the micro canonical and the canonical ensembles coincide.
- 108. Discuss the energy and particle number fluctuations in grand canonical ensemble.
- 109. Discuss the extend of fluctuation in energy and number density in Grand canonical ensemble and deduce the empirical relation for rms fluctuations in number density and energy. How is their value during phase transition?
- 110. Obtain the general expression for the distribution function for three kinds of statistics
- 111. Discuss the statistics of the microcanonical, canonical and grand canonical ensembles quantum mechanically.
- 112. Explain the statistics of micro canonical ensemble quantum mechanically.
- 113. Explain an ideal gas in Micro Canonical ensemble.
- 114. Explain an ideal gas in Canonical ensemble and Derive the expression for the most probable number of particles for energy level.
- 115. Explain the Bose Einstein's condensation.
- 116. Derive the equation of state and number density of Bose systems in terms of g functions. Hence work out the thermodynamic functions.
- 117. Outline the thermodynamics of an ideal Bose gas and derive the condition for the onset of Bose-Einstein condensation.
- 118. What is mean by Bose Einstein condensation and the derive its onset condition.
- 119. Explain Bose Einstein condensation. Deduce the expression for critical temperature.
- 120. Outline the thermodynamics of an ideal Bose gas and derive the condition for the onset of Bose-Einstein condition.
- 121. Discuss thermodynamics of Ideal Bose gas.
- 122. Explain Debye theory of specific heat. How does it differ from Einstein's theory of specific heat?
- 123. Explain Einstein's and Debye's model for sound waves.

- 124. Obtain Debye's law for phonons.
- 125. Discuss in detail the thermodynamic behaviour of ideal Fermi gas.
- 126. Discuss thermodynamics of Fermi Bose gas.
- 127. Discuss the magnetic behaviour of an ideal fermi gas
- 128. Discuss the Pauli paramagnetism in detail by considering it as highly degenerate Fermi gas.
- 129. Discuss the Pauli paramagnetism in detai
- 130. Work out the temperature dependent paramagnetic susceptibility of ideal Fermi systems.
- 131. Show that absolute zero, paramagnetic susceptibility is density dependent and temperature independent for a fermi gas.
- 132. Discuss the classical treatment of paramagnetism and derive Cure's law.
- 133. Obtain the general expression for paramagnetic susceptibility of ideal fermi gas. Discuss the nature of susceptibility at low and high temperature.
- 134. Show that for a fermi gas diamagnetics susceptibility obeys Curie's law at high temperature.
- 135. Discuss the Landau diamagnetism in detail.

C Part

- 136. Calculate the number of micro-states for four particles having a total energy of 6 E, the energy levels are equally spaced.
- 137. Show that the pressure of a non-relativistic gas is 2/3 of its energy density.
- 138. Give the physical meaning of the parameters β , η and ξ .
- 139. Write the recipe for deriving thermodynamics from a statistical beginning.
- 140. Find the number of microstate accessible to a system of particles of mass m each occupying a volume V in the energy interval E and E+dE.
- 141. What does the parameter β , η and ξ symbolizes? Give the physical meaning of the parameters
- 142. Explain the recipe for deriving thermodynamics from a statistical beginning.
- 143. What is mean by Gibbs paradox how it is resolved.
- 144. Explain Gibb's paradox. How it is resolved?
- 145. Show that $\ln \Box = \ln \Sigma$ for a classical ideal gas.
- 146. Write the Sackur-Tetrode equation and hence obtain an expression for chemical potential of classical ideal gas in terms of N, V and T. Check whether it is intensive or extensive.
- 147. State and prove Liouville's theorem.
- 148. Explain the possible solutions of ρ for satisfying the Liouville's theorem for a system in equilibrium i.e $[\rho, H] = 0$
- 149. Show that entropy of a collection of classical harmonic oscillators is always extensive
- 150. Prove that the phase space area equivalent to one Eigen state of a linear harmonic oscillator is h.
- 151. Show that the phase trajectory of a harmonic oscillator is an ellipse.
- 152. Calculate the Helmholtz free energy and entropy of a system of N classical harmonic oscillators.
- 153. Consider a system of independent 1-D quantum harmonic oscillators in canonical ensemble with the partition function QN (β) = $e^{-}N\beta\hbar\omega/2$ (1 $e^{-}\beta\hbar\omega$)⁻N.Calculate the Helmholtz free energy, entropy, pressure and

chemical potential for this system.

- 154. Derive the canonical partition function of a classical ideal gas consisting of N identical monatomic molecules confined to a volume V and in equilibrium at temperature T and hence obtain an expression for its entropy.
- 155. For a system of independent non interacting one-dimensional quantum harmonic oscillators, what is the value of the Helmholtz free energy per oscillator, in the limit temperature tends to zero?
- 156. How can an essential link be provided between the thermodynamics of a given system and the statistics of the corresponding grand canonical ensemble.
- 157. Obtain the specific heat capacity for ideal gas in grand canonical ensemble Cv. The single particle canonical partition function has the form $Q_1(V, T) = V f(T)$, f(T) where is a function of temperature alone.
- 158. Prove that the expectation value of a physical quantity G, $\langle G \rangle = \frac{Tr(\hat{\rho}\hat{G})}{Tr(\hat{\rho})}$
- 159. Obtain the equation of motion for the density matrix in quantum statics.
- 160. Derive the expression for the expectation value of a physical quantity G
- 161. Show that $\rho 2 = \rho$ for microcanonical ensemble, where ρ is the density operator.

^{162.} Show that $\langle \sigma_z \rangle = tanh(\beta \mu_B B)$

- 163. Obtain the general expression for the distribution function of Bose Einstein and fermi dirac statistics.
- 164. Consider a system of 3 fermions which can occupy any of the 4 available energy states with equal probability. What is the entropy of the system?
- 165. Show that $\overline{N} = \sum_{\varepsilon} \langle n_{\varepsilon} \rangle$ and $\overline{E} = \sum_{\varepsilon} \langle n_{\varepsilon} \rangle \varepsilon$
- 166. Explain briefly an ideal gas in micro canonical mechanical ensemble.
- 167. Show that the q potential, $q = a^{-1} \sum_i g_i \ln(1 + a e^{-\alpha \beta \varepsilon_i})$ and hence derive PV = NKT
- 168. Explain briefly an ideal gas in other quantum-mechanical ensembles.
- 169. Show that the most probable no of particles per energy level $\frac{n_i^*}{g_i} = \frac{1}{e^{\alpha + \beta \varepsilon_i + a}}$
- 170. Explain the statistics of occupation numbers.
- 171. Discuss the statistics of occupational number for the three distributions and show that they converge the same value in the classical limit.
- 172. Show that $\frac{N-N_0}{KT} = \frac{g_{3/2}(z)}{\lambda^3}$
- 173. Find the Internal energy and specific heat of a Bose system of N particles
- 174. $\frac{p}{KT} = \frac{g_{5/2}(z)}{\lambda^{8}}$
- 175. Derive the condition for onset of Bose Einstein condensation.
- 176. Explain Bose Einstein condensation and derive the onset conditions.

- 177. Derive the energy density of the black body radiation.
- 178. Show that radiation pressure exerted by the photons is equal to one third of its energy density.
- 179. Show that for a black body radiation pressure is proportional to the fourth power of temperature.
- 180. Show that radiation pressure exerted by the photons is equal to one third of its energy density.
- 181. Discuss the thermodynamics of the Black body radiation briefly
- 182. Show that radiation pressure is one third the energy density of a black body radiation.
- 183. Explain Debye's theory of specific heat.
- 184. Obtain briefly the Debye's law for phonons
- 185. Show that a system of phonons obeys T^3 law at low temperatures.
- 186. Explain Einesteins theory of Specific heat
- 187. Show that for Fermi gas at T= 0 K the average energy per particle not zero but $\frac{3}{5}$ E_E where E_F the fermi energy.
- 188. Discuss the analytical study of Fermi systems at finite but low temperatures.
- 189. Derive the expression for the Ground state pressere of fermi system.
- 190. Derive the expression for the Helmholtz free energy and entropy of the Fermi gas.
- 191. Derive the expression for the internal energy and Specific heat of the Fermi gas.
- 192. Discuss the thermodynamics of Fermi gas in non degenerate, degenerate and completely degenerate case.
- 193. Atomic weight of Lithium is 6.94 and density 0.53gm/cm3. Calculate Fermi energy and Fermi temperature of electrons.
- 194. Fermi systems are quite lively at absolute zero. Explain this statement.
- 195. Discuss the nature of Fermi gas at finite but low temperatures and arrive at the equation of state. Show that the specific heat capacity is proportional to the temperature.
- 196. Distinguish between non degenerate, degenerate and completely generate fermi gas.
- 197. Derive the expression for the Fermi energy and zero point energy of the system.
- 198. 'Even at absolute zero, the Fermi system is quite live'. Explain
- 199. Atomic weight of Lithium is 6.94 and density 0.53 g/cm³. Calculate Fermi energy and Fermi temperature of electrons.
- 200. Based on Landau's assumptions deduce the expression of Magnetic moment of a charged particle in an external magnetic field at high temperature
- 201. Obtain an expression for total energy of electrons at zero Kelvin.
- 202. Find an expression for Fermi energy of a two dimensional electron gas.
- 203. Derive the expression for Susceptibility holds for all temperatures.
- 204. Explain Pauli Paramagnetism.
- 205. Show that at low temperature paramagnetic susceptibility depends on density and independent of temperature.
- 206. Show that magnetisation $M = -\overline{N}\mu_{eff}L(x)$

- 207. Consider a system of 3 fermions which can occupy any of the 4 available energy states with equal probability. What is the entropy of the system?
- 208. Discuss the magnetic behaviour of an ideal fermi gas.
- 209. Discuss the magnetic behaviour of an ideal fermi gas briefly.
- 210. Derive the expression for the magnetic moment of the gas
- 211. Show that at low temperature diamagnetic susceptibility depends on density and independent of temperature
- 212. Discuss the Electron gas in a metals.
- 213. Explain Landau diamagnetism.
- 214. Derive the expression for the heat capacity of the electron gas.
- 215. Discuss the Specific heat of the electron gas.

D Part

E Part