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C 83626 (Pages : 3) Name..............................

Reg. No..............................

SECOND SEMESTER M.Sc. DEGREE (CUCSS) EXAMINATION, JUNE 2015

Mathematics

MT 2C 09—PDE AND INTEGRAL EQUATIONS

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries a weightage of 1.

1. Find the partial differential equation by eliminating the arbitrary function F from the equation $F(x+y, x-\sqrt{z}) = 0$.

2. Show that $z = ax + \left(\frac{y}{a}\right) + b$ is a complete integral of $pq = 1$.

3. Show that the equations $xp - yq - x = 0$ and $x^2 p + q - xy = 0$ are compatible.

4. State the Cauchy problem for the first order partial differential equation $f(x, y, z, p, q) = 0$.

5. What is characteristic strip ?

6. Define domain of dependence in the case of a one dimensional wave equation.

7. State the Neumann problem for the upper half plane.

8. Show that the solution to the Dirichlet problem is stable.

9. Show that the equation $(\sin^2 x)\, u_{xx} + 2(\cos x)\, u_{xy} - u_{yy} = 0$ is hyperbolic and find the characteristics.

10. What is Riemann function ?

11. Differentiate between Fredholm and Voltera integral equations.

12. Show that $y(x) = \int_0^x (x-\xi)\, F(\xi)\, d\xi + y'_0 x + y_0$ satisfies the differential equation $y'' = F(x)$ and the initial conditions $y(0) = y_0$ and $y'(0) = y'_0$.

Turn over

13. Show that the Kernel $K(x, \xi) = 1 + 3x\xi$ has a double characteristic number associated with (–1,
with two independent characteristic functions.

14. Determine the resolvent Kernel associated with $K(x, \xi) = \cos(x, \xi)$ in $(0, 2\pi)$, in the form of a po
series in λ.

(14 × 1 = 14 weight

Part B

Answer any **seven** *questions.*
Each question carries a weightage of 2.

15. Find the general integral of $(y+1)\,p + (x+1)\,q = z$.

16. Explain Charpit's method to find a complete integral of a first order partial differential equa
$f(x, y, z, p, q) = 0$.

17. Find a complete integral of the equation $p^2 x + q^2 y = z$, by Jacobi's method.

18. Solve the initial value problem for the quasi-linear equation $zz_x + z_y = 1$ with the initial condi
$x = s,\ y = s,\ z = \frac{1}{2}s$ for $0 \le s \le 1$.

19. Solve :

$$\frac{\partial^2 y}{\partial x^2} = c^2 \frac{\partial^2 y}{\partial t^2},\ 0 < x < \infty,\ t > 0$$
$$y(x, 0) = u(x),\ y_t(x, 0) = v(x),\ x \ge 0.$$

20. Solve the Dirichlet problem for a circle by choosing a suitable Green's function.

21. Solve :

$$u_t = u_{xx},\ 0 < x < l,\ t > 0$$
$$u(0, t) = u(l, t) = 0$$
$$u(x, 0) = x(l - x),\ 0 \le x \le l.$$

22. Transform the problem $\frac{d^2 y}{dx^2} + xy = 1,\ y(0) = y(1) = 0$ to the integral equation :

$$y(x) = \int_0^1 G(x, \xi)\,\xi\, y(\xi)\, d\xi - \frac{1}{2}x(1-x),$$ where $G(x, \xi) = x(1-\xi)$ when $x < \xi$ and $G(x, \xi) = \xi$
when $x > \xi$.

23. Prove that the equation $y(x) = \frac{1}{\pi} \int_0^{2\pi} \sin(x + \xi)\, y(\xi)\, d\xi + F(x)$ possesses no solution when $F(x) = x$, but it has infinitely many solutions when $F(x) = 1$.

24. Solve the Fredhom equation by iterative method :

$$y(x) = \lambda \int_0^1 (x + \xi)\, y(\xi)\, d\xi + 1.$$

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*

Each question carries a weightage of 4.

25. Show that the Pfaffian differential equation $yzdx + (x^2y - zx)\, dy + (x^2z - xy)\, dz = 0$ is integrable and find the corresponding integral.

26. Using the method of characteristics, find an integral surface of $p^2x + qy - z = 0$ containing the initial line $y = 1, x + z = 0$.

27. Describe the classification of second order partial differential equation :

$$R(x, y) \frac{\partial^2 u}{\partial x^2} + S(x, y) \frac{\partial^2 u}{\partial x \partial y} + T(x, y) \frac{\partial^2 u}{\partial y^2} + g(x, y, u, u_x, u_y) = 0,$$ where R, S and T are continuous functions of x and y possessing continuous derivatives.

28. Show that any solution of the equation $y(x) = \lambda \int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi + F(x)$ can be expressed as the sum of $F(x)$ and some linear combination of the characteristic functions.

(2 × 4 = 8 weightage)