16P201 (Pages:2) Name.....................

Reg.No.....................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY-2017

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15PMT2C06-ALGEBRA-II

(Mathematics)

(2015 Admission Onwards)

Time:Three Hours Maximum:36 Weightage

Part A

Answer **all** questions

Each question carries 1 weightage.

1. Let E be an extension field of a field F. Let $\propto \in E$ be algebraic over F. Define irreducible polynomial for $\propto$ over F.
2. Prove that the field **C** of complex numbers is an algebraically closed.
3. Check whether $\propto = \sqrt{1+\sqrt{3}}$ is algebraic over Q.
4. Prove that trisecting the angle is impossible.
5. Find the primitive 10 th root of unity in Z_{11}.
6. Find the splitting field of $x^3 - 2$ over Q.
7. Find all conjugates in C of $\sqrt{1+\sqrt{2}}$ over Q.
8. Let K be a finite normal extension of F and let E be an extension of F, where $F \leq E \leq K \leq \bar{F}$. Prove that K is a finite normal extension of E.
9. Find $\Phi_8(x)$ over Q.
10. Find $\{Q(\sqrt{2}, \sqrt{3}) : Q\}$
11. State isomorphism extension theorem.
12. Describe the group of the polynomial of $x^3 - 1$ over Q.
13. Show that the polynomial $x^5 - 2$ is solvable by radicals over Q.
14. Determine whether there exist a finite field having 68921number of elements.

(14×1 = 14 weightage)

Part B

Answer **any seven** questions

Each question carries 2 weightage.

15. Describe the field $Z_2[x]/<x^2+x+1>$.
16. Prove that an ideal $< p(x) > \neq \{0\}$ of $F[x]$ is maximal iff $p(x)$ is irreducible over F.
17. If E is a finite extension field of a field F and K is a finite extension field of E, Prove that K is a finite extension of F and $[K:F] = [K:E][E:F]$.

18. If F is a field of prime characteristic p with algebraic closure $\overline{F}$, prove that $x^{p^n} - x$ has p^n distinct zeros in $\overline{F}$.

19. If $\sqrt{a} + \sqrt{b} \neq 0$, Show that $Q(\sqrt{a} + \sqrt{b}) = Q(\sqrt{a}, \sqrt{b})$, $\forall$ a, b $\in$ Q.

20. Describe the group $G(Q(\sqrt{2}, \sqrt{3})/Q)$

21. Let $\overline{F}$ and $\overline{F}'$ be two algebraic closures of F. Prove that $\overline{F}$ is isomorphic to $\overline{F}'$ under an isomorphism leaving each element of F fixed.

22. Prove that the Galois group of the n^{th} cyclotomic extension of Q has $\emptyset(n)$ elements and is isomorphic to the group consisting of the positive integers less than n and relatively prime to n under multiplication modulo n.

23. If $E \leq \overline{F}$ is a splitting field over F, Prove that every irreducible polynomial in F[x] having a zero in E splits in E.

24. Let K be a splitting field of x^4+1 over Q. Show that $G(K/Q)$ is of order 4.

(7×2 = 14 weightage)

Part C

Answer **any two** questions
Each question carries 4 weightage.

25. Let R be a commutative ring with unity. Prove that M is a maximal ideal of R if and only if R/M is field.

26. State and prove conjugation isomorphism theorem

27. Prove that a finite separable extension of a field is a simple extension.

28. Let R be a commutative ring with unity then prove that M is a maximal ideal of R if and only if R/M is a field.

(2×4 = 8 weightage)

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